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Identifying diffraction effects in measured reflectances

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Abstract
There are two different physical models connecting the micro-geometry of a surface and its physical reflectance properties (BRDF). The first, Cook-Torrance, assumes geometrical optics*: light is reflected and masked by the micro-facets. In this model, the BRDF depends on the probability distribution of micro-facets normals. The second, Church-Takacs, assumes* diffraction *by the micro-geometry. In this model, the BRDF depends on the power spectral distribution of the surface height. Measured reflectance have been fitted to either model but results are not entirely satisfying. In this paper, we assume that both models are valid in BRDFs, but correspond to different areas in parametric space. We present a simple test to classify, locally, parts of the BRDF into the Cook-Torrance model or the diffraction model. The separation makes it easier to fit models to measured BRDFs.*

1. Introduction

The reflectance properties of a material are usually connected to the micro-geometry of its surface. Several models can predict the overall reflectance function (BRDF) from a physical description of the properties of the surface.

The most commonly used in Computer Graphics is the Cook-Torrance model [**?**]. Assuming that light interacts with the surface following optical geometry, it provides a compact model that depends only on the surface roughness. The main parameter is the probability distribution of micro-facet normals, $D(\theta_h)$.

Another model [**?**] assumes that the surface micro-geometry diffracts the incoming light. It is widely used in the optical engineering community and was recently introduced to the computer graphics community by Löw *et al.* [**?**].

Both models have been used to fit measured reflectance [**?**, **?**, **?**]. These are not entirely satisfying. The most numerically accurate fits required relaxing at least one physical rule: Bagher *et al.* [**?**] used a different distribution of micro-facets for each color channel; Löw *et al.* [**?**] removed the wavelength dependency in their fits. The common solution of adding several reflectance lobes to improve the quality of the fit also has no physical basis: in multi-layered materials, the interactions between the different layers is more complex than simply adding their reflectances [**?**].

In this paper, we assume that both the Cook-Torrance model and the diffraction model are active, at the same time, in the way a material interacts with light. Looking at measured reflectance, we want to test which of the two models is predominant, and use this information for better fitting.

We present a simple test, based on partial derivatives of the measured reflectance function. This test identifies areas in parameter space where diffraction is likely to be the main explanation for the reflectance. Experimental results show that diffraction effects correspond mostly to wide-angle reflection. This knowledge can then be incorporated to fit parametric models to measured BRDFs.

2. Previous work

2.1. Measured reflectances

Matusik *et al.* [**?**] measured and released reflectance properties for a large range of materials. We use their database in our tests. Ngan *et al.* [**?**] have fitted parametric BRDF models to this measured data. They found the best fits for the He, Cook-Torrance and Lafortune models.

Ashikhmin and Premože [**?**] approximated measured BRDFs using back-scattering: if input and output directions are equal, the entire BRDF can be expressed as a function of the half-vector. By storing this function, they get a compact BRDF model, that fits measured data very well.

2.2. Cook-Torrance model

The Cook-Torrance model [?] assumes that the micro-geometry of the surface is made of planar micro-facets. These micro-facets reflect the incoming light, but also block incoming and outgoing light from grazing angles. The full BRDF model is expressed as a product of three functions:

$$p_s(\boldsymbol{i}, \boldsymbol{o}) = \rho_s \frac{F(\boldsymbol{i}, \boldsymbol{h})G(\boldsymbol{i}, \boldsymbol{o})D(\theta_h)}{4(\boldsymbol{i} \cdot \boldsymbol{n})(\boldsymbol{o} \cdot \boldsymbol{n})} \quad (1)$$

D is the probability distribution for the orientation of micro-facet normals. F is the Fresnel reflection coefficient for each micro-facet, G is the masking and shadowing term, expressing how much of the incoming and outgoing light is masked by local geometry. It is computed from D by a double indefinite integration [?, ?, ?].

D is the main parameter in the Cook-Torrance model. Early work used a gaussian distribution, which was not a good fit with measured data. Trowbridge and Reitz [?] and Walter *et al.* [?] introduced the TR/GGX distribution, providing a better fit with measured data. Bagher *et al.* [?] used a SGD distribution for an even better fit with measured data. They found that using a different distribution for each color channel improved the quality of the fit as well as the convergence speed.

2.3. Diffraction model

The diffraction model is widely used in the optical engineering community [?]. Incoming light is diffracted by the micro-geometry of the surface. The BRDF model has one main parameter: S_z, the power spectral density of the surface height fluctuations:

$$p_w(\boldsymbol{i}, \boldsymbol{o}) = F(\boldsymbol{i}, \boldsymbol{h})S_z(f) \quad (2)$$

f encodes the wavelength dependency; it is equal to $||\boldsymbol{n} \times (\boldsymbol{i} + \boldsymbol{o})||$, divided by the wavelength λ. F is the Fresnel term, similar to the term used in equation 1.

Löw *et al.* [?] introduced the diffraction model to the computer graphics community. They show that it provides a good explanation for some behaviour of measured BRDFs, and a good approximation for measured data. However, they removed the explicit wavelength dependency of the diffraction effect in their model.

3. Identifying diffraction effects

3.1. Main hypothesis

Our main observation is that the Cook-Torrance model and the diffraction model are not mutually exclusive. The same micro-facet can both reflect incoming light and diffract it, along its edges. The BRDF of the material is then a sum of both models, along with a diffuse component:

$$p(\boldsymbol{i}, \boldsymbol{o}) = \frac{\rho_d}{\pi} + \rho_s \frac{F(\boldsymbol{i}, \boldsymbol{h})G(\boldsymbol{i}, \boldsymbol{o})D(\theta_h)}{4(\boldsymbol{i} \cdot \boldsymbol{n})(\boldsymbol{o} \cdot \boldsymbol{n})} + \rho_w S_z(f) \quad (3)$$

We made the assumption that Fresnel effects are associated mainly to specular reflection, not diffraction.

The half-angle parametrization [?], $(\theta_h, \theta_d, \phi_d)$ is a convenient parametrization for BRDFs. We can express f in this parameterization: $f = \frac{2}{\lambda} \sin\theta_h \cos\theta_d$, and use this to express the full BRDF model:

$$p(\boldsymbol{i}, \boldsymbol{o}) = \frac{\rho_d}{\pi} + \rho_s \frac{F(\theta_d)GD(\theta_h)}{4(\boldsymbol{i} \cdot \boldsymbol{n})(\boldsymbol{o} \cdot \boldsymbol{n})} + \rho_w S_z \left(\frac{2}{\lambda} \sin\theta_h \cos\theta_d\right) \quad (4)$$

3.2. Derivatives-based test

Our goal is to identify areas in parameter space where diffraction effects are predominant. In these places, the BRDF is determined mainly by $S_z(f)$:

$$p(\boldsymbol{i}, \boldsymbol{o}, \boldsymbol{n}) \approx \frac{\rho_d}{\pi} + \rho_w S_z(f) \quad (5)$$

In that case, the partial derivatives of the BRDF are connected to the derivative of S_z:

$$\nabla p = \rho_w \frac{\mathrm{d}S_z}{\mathrm{d}f} \nabla f \quad (6)$$

For this equation to be true, we must have: $\det(\nabla p, \nabla f) = 0$. This gives us a condition that must be verified if diffraction effects are dominant:

$$\det(\nabla p, \nabla f) = \frac{\partial p}{\partial \theta_h} \sin\theta_h \sin\theta_d + \frac{\partial p}{\partial \theta_d} \cos\theta_h \cos\theta_d = 0 \quad (7)$$

4. Results

4.1. Numerical computations

We computed partial derivatives on measured data using finite differences. Derivatives computed using the raw data are quite noisy and unsuitable for testing using Equation 7. We begin by averaging the data over ϕ_d to reduce the noise. We also store the variance over ϕ_d, and ignore areas with high variance, where the average is a poor representation of the data. We then compute our test function, $\det(\nabla p, \nabla f)$.

4.2. Mapping the results

We have computed the value of $\det(\nabla p, \nabla f)$ for all BRDFs in the MERL database. Figure 1 displays the value of $\det(\nabla p, \nabla f)$ using a color ramp, along with the value of the BRDFs, for some representative materials. Diffraction effects are dominant for areas in blue and purple. Several things appear clearly:

- The separation between diffraction effects and other causes is clearly visible for specular-type BRDFs, such as metals and shiny plastics. For more diffuse BRDFs, such as woods and plastics, the separation is less marked.

- Diffraction effects, when present, correspond to wide-angle scattering, and lower values of the BRDF. These wide-angle scattering plays an important role in the visual aspect of the BRDF. It provides the color of the material outside of the specular peak.
- The specular peak, around $\theta_h = 0$, does not appear to correspond to diffraction effects.

4.3. Wavelength dependency

The diffraction model predicts the BRDF dependency on wavelength: they should depend on $f = \frac{2}{\lambda} \sin\theta_h \cos\theta_d$, up to a multiplicative constant. To validate this hypothesis, we plot BRDF values for sample points where $\det(\nabla p, \nabla f)$ is under a certain threshold (0.01). Figure 2(a) shows these BRDF values as a function of f for one material (alum-bronze). The behaviour appears to be as predicted by the theory: curves for the three channels appear to be very similar, once their x-axis has been scaled by λ. For other BRDFs, the superposition is not as perfect: there is a vertical scaling, corresponding to the specular color, and vertical translation corresponding to the diffuse color.

4.4. Possible interpretation

We present the following interpretation of our experimental results: the micro-geometry of the surface contributes to the BRDF through *both* reflection and diffraction. Reflection is explained by the Cook-Torrance lobe, with a distribution of normals independent of wavelength. It is responsible mostly for the specular peak. Diffraction contains wavelength dependency. It is responsible for wide-angle scattering. The two effects co-exist, but can be separated using partial derivatives.

The fact that two different effects are present could explain previous difficulties in fitting measured materials with a single model. It could also explain why previous research had to use complicated distributions such as SGD or ABC.

Preliminary experiments using this hypothesis show that fitting measured materials using a sum of diffraction and reflection models converges quickly and provides a good approximation (see Figure 2(b)).

4.5. Conclusion

Two different models describe the relationship between a surface micro-geometry and its overall appearance: one explains BRDF behaviour by specular reflection on the micro-facets, the other by diffraction by the surface geometry.

We have designed a test to identify potential areas where diffraction effects dominate. It appears that diffraction effects are present in most measured materials; they explain material behaviour for wide-angle scattering, but not for the specular lobe. It seems that both reflection and diffraction effects are present in measured materials.

In future work, we want to extensively test this hypothesis on a large set of measured materials. We also want to predict surface geometry based on the BRDF: the Cook-Torrance lobe gives the average shape of the micro-geometry, but not its size. The diffraction lobe gives the spatial frequency of the micro-geometry. Combining the two could provide a full model of the micro-geometry.

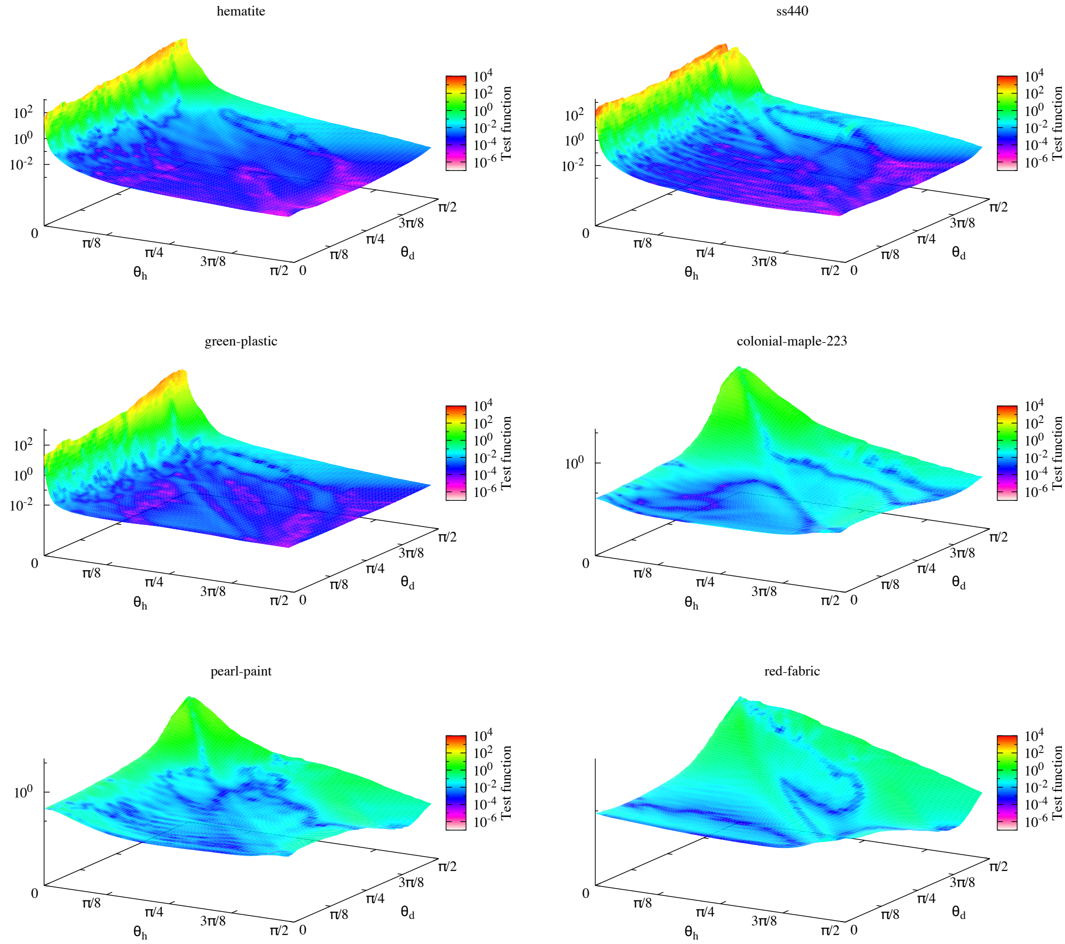
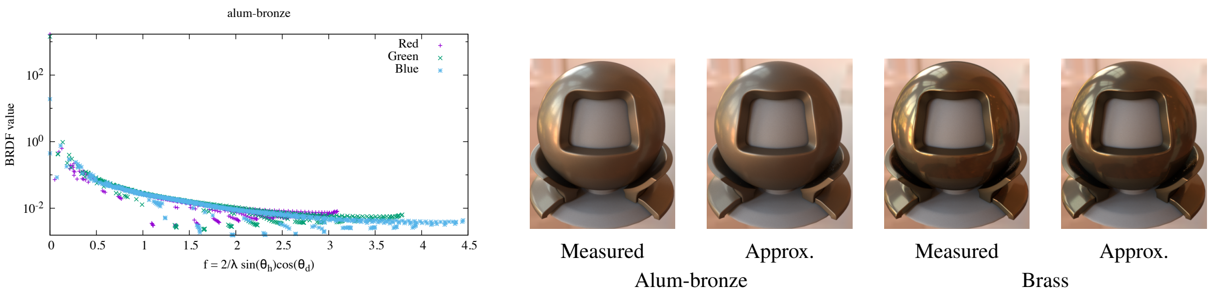


Figure 1: *Displaying the values of* $\det(\nabla p, \nabla f)$ *for representative BRDFs, in* (θ_h, θ_d) *space. z scale: BRDF intensity (in log space). Color scale: values of* $\det(\nabla p, \nabla f)$*. Diffraction effects are dominant for areas colored in blue and purple. For specular BRDFS (e.g. hematite, ss440, green-plastic) it is easy to separate this effect from other components. For more diffuse materials (e.g. colonial-maple-223, pearl-paint, red-fabric), the separation is less obvious.*



(a) BRDF values for sample points where diffraction effects are likely to be dominant ($\det(\nabla p, \nabla f) < 0.01$).

(b) Comparison between measured BRDFs and approximations using a combination of diffraction and Cook-Torrance models, with wavelength-dependence for diffraction.

Figure 2: *Using the knowledge about diffraction effects improves fitting of measured materials.*