Lights warping for enhanced surface depiction: Supplemental materials

1 Impact on rendering performances

The present study is done on a workstation with an Intel(R) Core(TM)2 Quad CPU at 2.4 GHz and with a NVIDIA GeForce 8800 GTX GPU. The image resolution is 800x600. Note that we do not study the influence of resolution on frame-rate, since most of the computation is done using fragment processing. This obviously leads to a linear complexity with the respect to the number of processed pixels.

Cost of local shape analysis

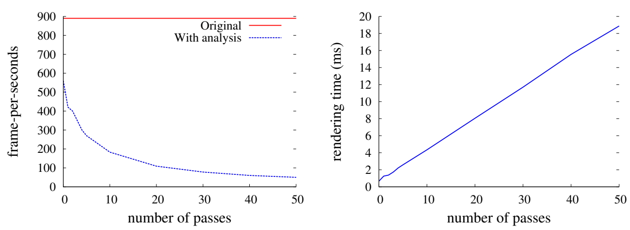

Figure 1: *Impact of local shape analysis on rendering times. Left: frame-rate loss due to increasing number of passes (i.e., diffusion scale). Right: processing cost for increasing number of passes.*

As shown in Section 4.4 of the paper, larger scales of the diffusion correspond to larger number of rendering passes. Intuitively, the rendering overhead of our shape analysis depends linearly on the number of passes (i.e., scale's size). Our study in Figure 1 confirms this assumption.The minimal overhead is 0.7 ms for an 800x600 image (a loss of 430 frame-per-seconds). Each diffusion step adds an overhead of 0.4 ms.

Cost of light warping

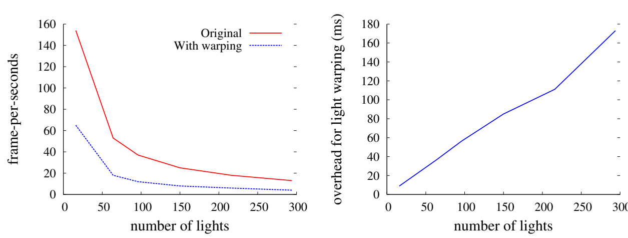

Figure 2: *Impact of light warping on rendering times. Left: frame-rate loss for to increasing number of light direction. Right: processing cost for increasing number of lights.*

As shown in Section 5 of the paper, for each pixel, our approach warps light directions using stereographic parameterization. Intuitively, the rendering overhead of our light warping depends linearly on the number of light directions used for the reflected radiance estimation. Our study for direct lighting in Figure 2 confirms again this assumption. There is no minimal overhead, but each new light direction adds 0.5 ms to the original rendering cost. Note that this cost is independent of the rendering technique, and is thus negligible for off-line global illumination. In our real-time implementation of direct lighting, and in our case study for an 800x600 image, this corresponds roughly to a half of the original frame-rate.

2 Jacobian for inverse warped reflection

To estimate reflected radiance using light warping we have formulated following equation in Section 6.1 of the paper:

$$L'(\boldsymbol{p} \rightarrow \boldsymbol{e}) = \int_\Omega \rho(\boldsymbol{e},\boldsymbol{\ell}) < \boldsymbol{n}\cdot\boldsymbol{\ell} > \; L_i(\boldsymbol{p} \leftarrow W(\boldsymbol{\ell}))\; d\boldsymbol{\ell}$$

We have also shown that, for some cases, it is more efficient to keep the lighting unchanged, resulting in the following equation with the jacobian $J = d\boldsymbol{\ell}/d\boldsymbol{\ell'}$:

$$L'(\boldsymbol{p} \rightarrow \boldsymbol{e}) = \int_\Omega \rho(\boldsymbol{e}, W^{-1}(\boldsymbol{\ell'})) < \boldsymbol{n}\cdot W^{-1}(\boldsymbol{\ell'}) > L_i(\boldsymbol{p} \leftarrow \boldsymbol{\ell'})\; J\; d\boldsymbol{\ell'}.$$

In the local frame $\{\boldsymbol{u},\boldsymbol{v},\boldsymbol{z}\}$, where $\boldsymbol{u},\boldsymbol{v}$ are the principal curvatures direction in image space (see Section 5 of the paper), the lighting directions can be noted classically as:

$$\boldsymbol{\ell} = \begin{pmatrix} \ell_u \\ \ell_v \\ \ell_z \end{pmatrix} = \begin{pmatrix} \cos(\phi)\sin(\theta) \\ \sin(\phi)\sin(\theta) \\ \cos(\theta) \end{pmatrix} \tag{1}$$

$$\boldsymbol{\ell'} = \begin{pmatrix} \ell'_u \\ \ell'_v \\ \ell'_z \end{pmatrix} = \begin{pmatrix} \cos(\phi')\sin(\theta') \\ \sin(\phi')\sin(\theta') \\ \cos(\theta') \end{pmatrix} \tag{2}$$

With this parameterization, $d\boldsymbol{\ell} = \sin(\theta) d\theta d\phi$.

Introducing the new variable $\tau = \tan(\theta/2)$, we can notice that $d\tau = 2/(1+\tau^2)d\theta$ and that $\sin(\theta) = 2\tau/(1+\tau^2)$. This leads to

$$d\boldsymbol{\ell} = \frac{4\tau}{(1+\tau^2)^2} d\tau d\phi.$$

The stereographic projection introduces two new variables: $a = \ell_u/(1+\ell_z) = \cos(\phi)\tau$ and $b = \ell_v/(1+\ell_z) = \sin(\phi)\tau$. Since $a^2 + b^2 = \tau^2$ and that $da\,db = \tau\, d\tau, d\phi$, this new substitution leads to

$$d\boldsymbol{\ell} = \frac{4}{(1+a^2+b^2)^2} da\,db. \tag{3}$$

In this stereographic space, light warping corresponds to the simple scalings $a' = \lambda_u a$ and $b' = \lambda_v b$:

$$d\boldsymbol{\ell} = \frac{4}{(1+a'^2/\lambda_u^2 + b'^2/\lambda_v^2)^2} \frac{da'}{\lambda_u} \frac{db'}{\lambda_v} \tag{4}$$

$$= \frac{4\lambda_u\lambda_v}{(\lambda_u^2\lambda_v^2 + \lambda_v^2 a'^2 + \lambda_u^2 b'^2)^2} da'\,db'. \tag{5}$$

Similarly to Equation 3, we can write

$$d\boldsymbol{\ell'} = \frac{4}{(1+a'^2+b'^2)^2} da'\,db', \tag{6}$$

Combining this result with Equation 5 leads to

$$d\boldsymbol{\ell} = \frac{\lambda_u^3\lambda_v^3(1+a'^2+b'^2)^2}{(\lambda_u^2\lambda_v^2 + \lambda_v^2 a'^2 + \lambda_u^2 b'^2)^2} d\boldsymbol{\ell'}. \tag{7}$$

Since a' and b' are the stereographic projection of the warped light direction $\boldsymbol{\ell'}$, $a' = \ell'_u/(1+\ell'_z)$ and $b = \ell'_v/(1+\ell'_z)$.

$$d\boldsymbol{\ell} = \frac{\lambda_u^3\lambda_v^3((1+\ell'_z)^2 + l_u'^2 + l_v'^2)^2}{(\lambda_u^2\lambda_v^2(1+\ell'_z)^2 + \lambda_v^2 l_u'^2 + \lambda_u^2 l_v'^2)^2} d\boldsymbol{\ell'} = J d\boldsymbol{\ell'}. \tag{8}$$

Since $1 - l_z'^2 = l_u'^2 + l_v'^2$, we finally get

$$J = \frac{4\lambda_u^3\lambda_v^3(1+\ell'_z)^2}{(\lambda_u^2\lambda_v^2(1+\ell'_z)^2 + \lambda_v^2 l_u'^2 + \lambda_u^2 l_v'^2)^2}$$