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Supplemental Material for Short Paper: Real-Time Importance Sampling of Dynamic Environment Maps

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1. Probability Density Function $\mathbf{pdf}_\mathrm{L}(\omega_i|f)$

The piecewise PDF associated with the tabulated CDF is computed as follow:

$$\mathrm{pdf}_\mathrm{L}(\boldsymbol{\omega}_i|f) = \frac{I_{ij}}{I_f}\frac{K}{4}(1+u_i^2+v_i^2)^{3/2} \tag{1}$$

where (u_i, v_i) are the face coordinates corresponding to direction $\boldsymbol{\omega}_i$ (values between $[-1,1]$), I_{ij} is the luminance of the corresponding pixel and, I_f is the sum of all pixels' luminance belonging to face f. K is the total number of pixels of the environment map face.

Intuitively, $4/K$ is the pixel size on the unit cube and $4(1+u_i^2+v_i^2)^{-3/2}/K$ is approximately its solid angle. This factor thus projects a unit cube onto a unit sphere.

The corresponding CDF is the expected piecewise linear function. This is demonstrated as follow. For pixel (k,l) on face f, the CDF is expressed as

$$\mathrm{CDF}_\mathrm{L}^{kl}(\boldsymbol{\omega}|f) = \sum_{i\le k}\sum_{j\le l}\int\int \mathrm{pdf}_\mathrm{L}(\boldsymbol{\omega}|f)\,\phi_{ij}(u,v)\,\mathrm{d}\boldsymbol{\omega}$$

where ϕ_{ij} is equal to 1 on pixel (i,j) and zero elsewhere. Introducing Equation 1 leads to

$$\mathrm{CDF}_\mathrm{L}^{kl}(\boldsymbol{\omega}|f) = \frac{1}{I_f}\frac{K}{4}\sum_{i\le k}\sum_{j\le k} I_{ij}\int\int (1+u^2+v^2)^{3/2}\,\phi_{ij}(u,v)\,\mathrm{d}\boldsymbol{\omega}$$

Since $\mathrm{d}\boldsymbol{\omega} = 1/(1+u^2+v^2)^{3/2}$, this simplifies to

$$\mathrm{CDF}_\mathrm{L}^{kl}(\boldsymbol{\omega}|f) = \frac{1}{I_f}\frac{K}{4}\sum_{i\le k}\sum_{j\le k} I_{ij}\int\int \phi_{ij}(u,v)\,\mathrm{d}u\mathrm{d}v$$

Since ϕ_{ij} is constant, the resulting function is therefore piecewise linear. Since $\int\int \phi_{ij}(u,v)\,\mathrm{d}u\mathrm{d}v$ is the area of a pixel that is, $4/K$, and finally since I_f is the sum of all I_{ij}, we obtain on pixel (k,l)

$$\mathrm{CDF}_\mathrm{L}^{kl} = \frac{\sum_{i\le k}\sum_{l\le k} I_{ij}}{\sum_i\sum_j I_{ij}}$$

which is exactly computed using prefix sum.

References

[STJ*04] Stumpfel J., Tchou C., Jones A., Hawkins T., Wenger A., Debevec P.: Direct HDR capture of the sun and sky. In *Proc. AFRIGRAPH '04* (2004), ACM, pp. 145–149.

Figure 1: *Comparisons of rendered images with (**Left**) the weight β_i and without (**Right**) where discontinuities are introduced. The cube map size is* $512 \times 512 \times 6$.

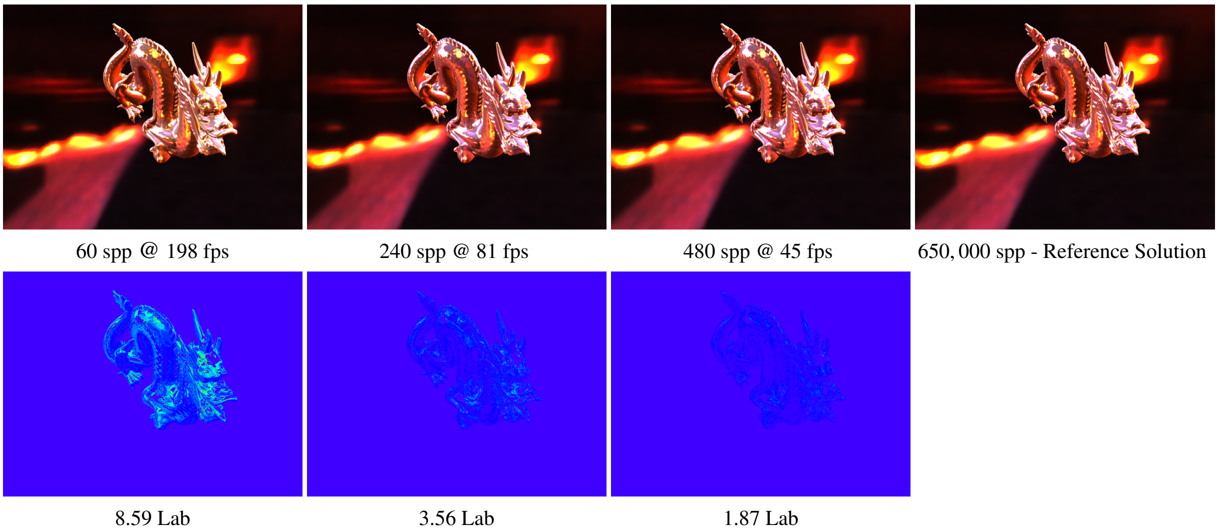

Figure 2: ***(First Row)*** *Qualitative comparisons between our technique when increasing the number of samples per pixel (spp) and a reference solution solution computed using* 655 000 *spp generated from a cosine-based hemisphere sampling scheme.* ***(Second Row)*** *Lab images showing the difference between our technique and the ground truth. The number below each image represent the mean Lab computed only for the dragon's pixels. As shown in the second row, our MIS technique converges toward the correct solution when increasing the number of samples. Furthermore, with 60 spp the resulting quality is already very good with a low Lab error and a high frame rate of 198 fps. The energy conserving Lafortune Phong BRDF shininess exponent is set to 120.*

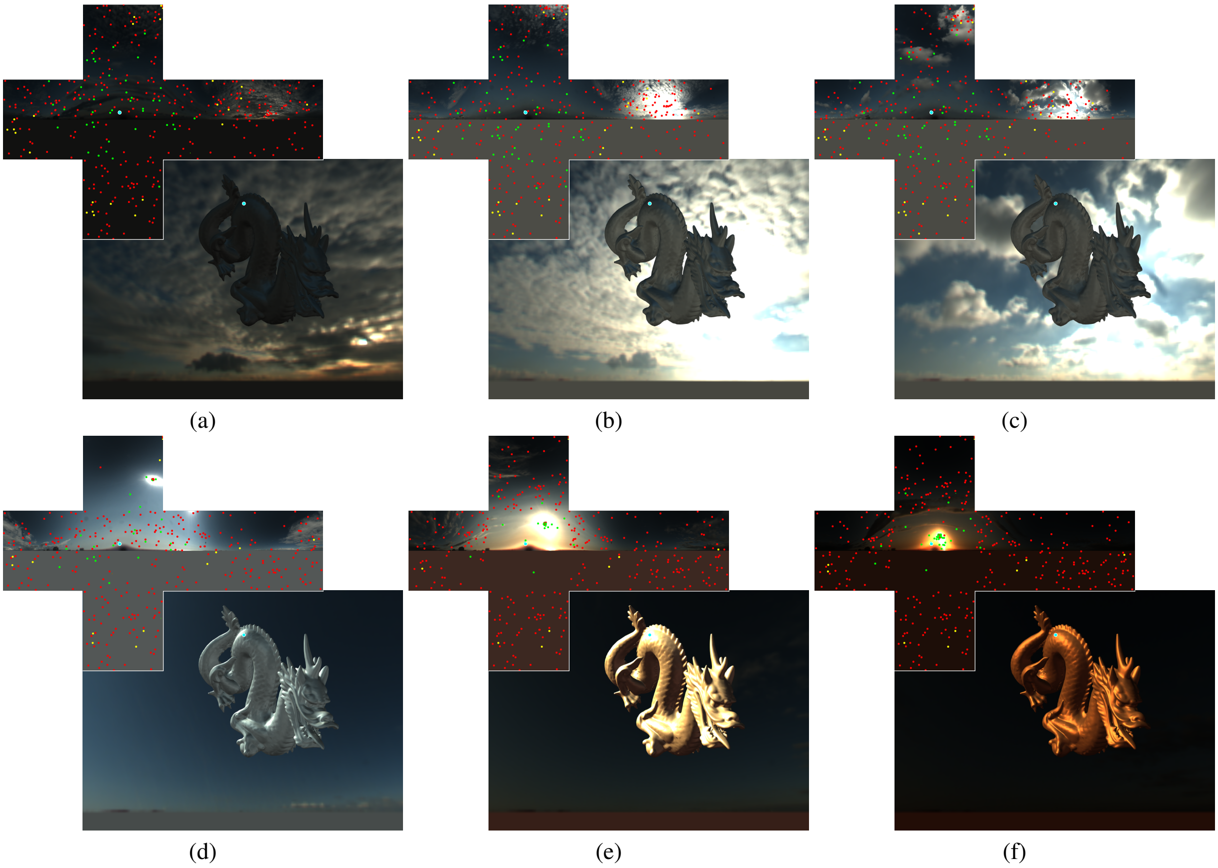

Figure 3: *Different images captured at different time of the day using free data from Stumpfel [STJ*04]. The upper left (resp. right) corner corresponds to an environment at sunrise (resp. sunset). For each image, we show where the light samples are located in the cross-map for the pixel located by the cyan dot on the dragon. In the inset image, the red dots correspond to pre-generated samples, the blue dot correspond to the direction of the normal ($\mathbf{n}$). The green and yellow dots are the samples selected by our technique. The green dots correspond to valid samples ($\langle \mathbf{n}, \boldsymbol{\omega} \rangle \geq 0$) whereas the yellow ones correspond to useless samples (with negative or null cosine factor). Observe how in images (b) to (d) our technique prevents generating samples on the sun because it is behind the shaded point. On the other side, in images (e) and (f) when the sun faces the shaded point the many samples are generated to faithfully represent its importance. We use only light sampling and the number of targeted samples (N_s) is 64. The average frame rate is 215 fps.*