Preface

The central theme of this book concerns Feynman-Kac path distributions, interacting particle systems, and genealogical tree based models. This recent theory has been stimulated from different directions including biology, physics, probability, and statistics, as well as from many branches in engineering science, such as signal processing, telecommunications, and network analysis. Over the last decade, this subject has matured in ways that make it more complete and beautiful to learn and to use. The objective of this book is to provide a detailed and self-contained discussion on these connections and the different aspects of this subject. Although particle methods and Feynman-Kac models owe their origins to physics and statistical mechanics, particularly to the kinetic theory of fluid and gases, this book can be read without any specific knowledge in these fields. I have tried to make this book accessible for senior undergraduate students having some familiarity with the theory of stochastic processes to advanced postgraduate students as well as researchers and engineers in mathematics, statistics, physics, biology and engineering.

I have also tried to give an "exposé" of the modern mathematical theory that is useful for the analysis of the asymptotic behavior of Feynman-Kac and particle models. Researchers and applied mathematicians will find a collection of modern techniques from various branches of probability and stochastic analysis, including convergence of empirical processes, fluctuation and large-deviation analysis, semigroup and martingale techniques and propagation of chaos, as well as the asymptotic stability and the concentration of measure-valued processes, functional inequalities, ergodic coefficients, and contractions of Markov operators and nonlinear semigroups.

Besides the mathematical analysis of Feynman-Kac distribution flows and interacting particle models, I have developed a rather large class of applications to specific models from various scientific disciplines. The practitioner will find a source of useful convergence estimates as well as a detailed list of concrete examples of particle approximations for real models, including restricted Markov chain simulations, random motions in absorbing media, spectral analysis of Schrödinger operators and Feynman-Kac semigroups, rare event analysis, Dirichlet boundary problems, nonlinear filtering problems, interacting Kalman-Bucy filters, directed polymer simulations, and interacting Metropolis type algorithms. While this diversity of application model areas is part of the charm of the particle theory of Feynman-Kac models the list topics above is not exhaustive and actually only reflects the tastes and interests of the author.

One objective in writing this book was to throw some new light on some interesting links between sometimes too disconnected physical, engineering, and mathematical domains. In this connection, I would like to thanks Springer-Verlag and the editorial board for the invitation to write a book on this theme. I undertook this project for two main reasons. First I felt that there was no accessible treatment on Feynman-Kac path models and their interacting particle approximation schemes. Second, the abstract concepts and the probability theory are now at a point where they provide a natural and unifying mathematical basis for a large class of heuristic-like Monte Carlo algorithms currently used in Bayesian statistics, engineering science, and in physics and biology since the beginning of 1950s. I also hope that practitioners as well as graduate students from Bayesian schools will find a great advantage in using the abstract Feynman-Kac and particle theory developed in this book, provided they overcome the fear of seeing integral operators rather than summations or integrals with respect to some density function. Besides its mathematical elegance, the abstract formulation is less "notationally consuming" and of great practical value. It gives a powerful applied tool to be used in modeling nonlinear estimation problems as well as in studying and developing interacting particle approximation models. I hope that these ideas will fruitfully serve the further development of the field and that their propagation will influence other new application areas.

The material in this book can serve as a basis for different types of advanced courses on probability. The first type, geared towards pure applications of particle methods, could be centered around the Feynman-Kac modeling techniques and their application model areas discussed in Chapters 2, 3, 11 and 12. To aid more detailed studies these lectures could be completed either with the presentation of one of the more application-related articles selected from the list of references or with a new application model. More theoretical types of courses could cover the material in Chapters 4 through 10. A semester-long course would cover the stability and the annealed properties of Feynman-Kac semigroups derived in Chapters 4, 5, and 6 (possibly excluding Chapter 5). There is also enough material in the book to support three other sequences of courses. These lectures would cover, respectively, propagation of chaos (Chapters 7 and 8), central limit theorems (Chapters 7 and 9) and large-deviation principles (Chapter 10).

A part of the material presented in this book is based on a series of lectures I delivered at the 24th Finnish Summer School on Probability Theory in Lahti in spring 2002 and that was arranged by the Finnish Graduate School in Stochastics and the Rolf Nevanlinna Institute. It is also partly based on a second-year graduate course on particle methods and nonlinear filtering I gave at the Operations Research and Financial Engineering department of Princeton University in fall 2001. An overview was presented in three one-hour lectures for the Symposium on Numerical Stochastics (April 1999) at the Fields Institute for Research in Mathematical Sciences (Toronto) and at the same time was presented at the University of Alberta, Edmonton, with the support of the Canadian Mathematics of Information Technology and Complex Systems project.

Some of the material developed in this book results from various fruitful collaborations with Frédéric Cérou, Dan Crisan, Donald Dawson, Arnaud Doucet, François Le Gland, Michael Kouritzin, Pascal Lézaud, Michel Ledoux, Terry Lyons, Philip Protter, Samuel Tindel, Frederi Viens, Tim Zajic, and particularly with Alice Guionnet, Jean Jacod, and Laurent Miclo. Most of the text also proposes many new contributions to the subject. The reader will find a series of new deeper studies of topics such as contraction properties of nonlinear semigroups, functional entropy inequalities, uniform and precise increasing propagation-of-chaos estimates, central limit and Berry-Esseen type theorems, large-deviation principles for strong topologies on path-distribution spaces, new branching and genealogical particle models, advanced Feynman-Kac modeling techniques, and a fairly new class of application modeling areas.

While continuous time models and their applications in physics and engineering sciences are discussed in Chapters 1 and 12, I have not hesitated to concentrate the exposition on discrete Feynman-Kac and particle models. The reasons are twofold:

First the analysis of discrete time models only requires a small prerequisite on Markov chains, while the study of continuous time models would have required different and specific knowledge on stochastic analysis, particularly on interacting jump models and Markov processes taking values in path spaces. Since one of the objectives was to prepare a text that was as self-contained as possible, a full presentation of both classes of models would have been too much digression.

The second reason is that apart from some mathematical technicalities the asymptotic analysis of continuous time models is in some sense more sophisticated but generally follows the same intuitions and the same line of argument as in the discrete time case. On the other hand, to my knowledge, various convergence theorems such as the uniform and increasing propagation of chaos, the Berry-Esseen estimates, and the strong large-deviation principles, presented respectively in Chapters 8, 9, and 10, remain nowadays open problems for continuous time models.

The reader interested in continuous time models can complete the study of this book following the four articles [94, 95, 96, 100]. For an introduction to interacting particle interpretation of continuous time Feynman-Kac models we recommend the review article on genetic type models [94] as well as [96] and [95, 100]. The last two referenced articles discuss both discrete and continuous time with applications to Schödinger generators, filtering problems and fixed points of integro-differential equations. I recommend [100] as a start for someone who has never studied the subject before. The article [95] provides a series of advanced lectures on simple central limit theorems and exponential estimates. Strong propagation-of-chaos results using coupling and semigroup techniques can be found in [93]. The series of articles above can also be completed by some studies on particle approximation of stochastic Feynman-Kac flows [63, 66, 64, 65] and their applications in the numerical solution of stochastic partial differential equations with non-linear potential [290]. I hope the former volume and the list of contributions to continuous time models above will guide the reader to understand the current state of the art on the topic and contribute to many interesting open problems.

I end this preface with a few words of advice to readers who are anxious about spending too much time on unnecessary immersions. The introduction of this book is an important step in entering into Feynman-Kac modeling and interacting-particle methods. The applications described in this opening section as well as in Chapters 11 and 12 should help the reader to find a concrete basis for going further through the mathematical aspects of this theory. A complete description on how the theory is applied in each application model area would of course require separate volumes with precise computer simulations and comparisons with different types of particle models and other existing algorithms. I have chosen to treat each subject in a rather short but self-contained way. Some applications are nowadays routine, and in this case I provide precise pointers to existing more application-related articles in the literature. Most applications also provide new insight on the theoretical and potential applications of Feynman-Kac and particle methods to statistical physics and engineering science. In this case the programming of these new particle algorithms is left to the reader. One natural path of "easy reading" will probably be to choose a familiar or attractive application area and to explore some selected parts of the book in terms of this choice. Nevertheless, this advice must not be taken too literally. To see the impact of genealogical tree-based particle methods, it is essential to understand the full force of Feynman-Kac modeling techniques on various research domains. Upon doing so, the reader will have a powerful weapon for the discovery of new Feynman-Kac interpretations and related particle numerical models. The principal challenge is to understand the theory and the branching particle models well enough to reduce them to practice.

I did not tried to avoid repetition and each chapter starts with an introduction connecting the results developed in earlier parts with the current analysis. With a few exceptions, this book is self-contained and each chapter can be read independently of the other ones. To get up to speed on Chapter 12 on applications, the reader is recommended to start with Chapters 2 and 3, which contain the main concepts on Feynman-Kac modeling and interacting processes. Chapter 11 should also not be skipped since it contains a series of recipes on particle models to be combined with one another and applied in each application model area. In general, I did not give references in the text but in the introduction at the beginning of each chapter. I already apologize for possible errors or for references that have been omitted due to the lack of accurate information.

Finally I would like to express my gratitude to the Centre National de la Recherche Scientifique (CNRS) which gave me the freedom and the opportunity to undertake this project and the Université Paul Sabatier of Toulouse. I am also grateful to the University of Melbourne and Purdue University as well as to Princeton University, where part of this project was developed. Last but not least, I would like to extend my thanks to John Kimmel for his precious editorial assistance as well as for his encouragements during these last two years.

Toulouse, France
September, 2003

Pierre Del Moral