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Madame:

I am attaching my report on the work of Pierre Del Moral in response to your request of 22 March, 2002.

Sincerely yours,

Donald Dawson

Donald Dawson
Professor Emeritus and
Distinguished Research Professor

Report on *CONTRIBUTIONS A L'ESTIMATION NON LINEAIRE ET AUX SYSTEMES DE PARTICULES EN INTERACTION*

by

Pierre DEL MORAL

The work outlined in this document is the outcome of a systematic research program to develop a mathematical framework and analytical and probabilistic tools to address the central problems of filtering/signal processing and optimal control. The latter subjects are of great importance in engineering science and technology with roots going back to the seminal contributions of Kolmogorov, Wiener, Bellman, Bucy, Kalman, Kushner, Stratonovich, Zakai and others. Over the past thirty years the subject has become a mature mathematical science and even more recently advances in computer technology have stimulated important synergism between the mathematical theory and the practical numerical implementation for a variety of engineering problems. The "curse of dimensionality" presents a serious challenge to applying the theoretical ideas to realistic problems. In response to these challenges a variety of mathematical approaches have been developed. The work of Pierre Del Moral brings to bear on these problems powerful ideas and tools of approximating interacting particle systems, Feynman-Kac methods, empirical processes, measure-valued processes optimization theory and large deviation theory.

This approach to these problems has wide applicability as represented in the eight papers in this work. The synthesis article and eight papers develop important methodology and application of these ideas and methods under headings:

1. Feynman-Kac formulae and interacting particle systems.
2. Genetic algorithms and global optimization.
3. Contraction properties of Markov operators and non-linear semigroups.
4. Idempotent probability calculus.

and I will comment separately on the contributions to each of these.

The review paper *Branching and Interacting Particle Systems Approximations of Feynman-Kac Formulae with Applications to Non-Linear Filtering* (Del Moral and Miclo) is a survey of an important research program developed in recent years by Del Moral, Guionnet, Crisan , Lyons and others. Del Moral has been one of the principal innovators of this development. The underlying idea is to use interacting particle systems and their empirical measures to obtain numerical approximations of Feynman-Kac formulae. A key part of the program is the study of the limits as the number of particles goes to infinity: this includes almost sure convergence, central limit theorems and Donsker's Theorem for particle density profiles. Special attention is also given to large deviations results for empirical measures on path space. Two classes of particle systems of interest are branching particle systems and mutation-selection particle systems

From: Donald A Dawson To: Laboratoire de Stat et Prob Date: 27/04/02 Time: 11:10:54 PM Page 3 of 5

in both discrete and continuous time. The main result of the paper is a method to design interacting particle systems to approximate unnormalized Feynman-Kac stochastic flows. Of note are the results which give exponential bounds on deviations from the limiting system.

The use of probabilistic algorithms for global optimization in complex landscapes is one of the important developments of the past twenty years. The idea of simulated annealing has its roots in statistical physics and the idea of genetic algorithms arises from the biological mechanisms of mutation-selection and recombination. The mathematical analysis of simulated annealing has been developed by a number of authors since the mid-1980's. The rigorous mathematical study of genetic algorithms is more recent. The paper *On the Convergence and Applications of Generalized Simulated Annealing (Del Moral and L. Miclo)* presents a general Markov process framework for the study of the convergence of generalized simulated annealing algorithms with non homogeneous potentials. In particular they construct a class of stochastic algorithms which converge to global extrema of integral potentials. They introduce new methods of analysis using a semigroup approach based on relative entropy and log Sobolev inequalities and an analysis of the extrema of a virtual energy function. These results are applied to obtain a basic convergence theorem for a class of genetic algorithms. An interesting and innovative feature of this method is a precise study of good or bad paths. These ideas can be applied to produce adaptive approximation schemes in nonlinear filtering theory. In *Particle approximations of Lyapunov exponents connected to Schrödinger operators and Feynman-Kac semigroups*. these ideas are used to study the Lyapunov exponent of a Feynman-Kac semigroup in terms of a fixed point of the evolution semigroup and estimates of the decay to equilibrium. In this setting approximating genetic particle systems are then analyzed.

The paper *Genealogies and Increasing Propagation of Chaos for Feynman-Kac and Genetic Models* is an important step in developing the ideas of historical processes and genealogies to the interacting particle systems described above to study Feynman-Kac formulae on path space. The main results describe the asymptotic behavior of the empirical measures of genetic algorithms approximating Feynman-Kac formulae. In the non-linear filtering context, for example, this makes possible the approximation of the time evolution of target trajectories in a tracking problem. In addition strong propagation of chaos results under mild conditions on the fitness functions and mutation transitions are obtained in which the deviation from the limit process is described by entropy estimates on path space over growing time intervals as the number of particles increases and also the Zolotarev $\mathcal{F}_n$ distance over an increasing number of bounded measurable functions (again as the number of particles goes to infinity).

There is a long history of interplay between analytical and probabilistic aspects of Feynman-Kac semigroups. In *On the Stability of Non Linear Feynman-Kac Semigroups* (Del Moral and L. Miclo) the authors obtain rates of convergence for non-linear Feynman-Kac semigroups. An interesting result is obtained which gives conditions under which a Markov process with killing conditioned

From: Donald A Dawson To: Laboratoire de Stat et Prob Date: 27/04/02 Time: 11:10:54 PM Page 4 of 5

on non-extinction forgets exponentially fast its initial condition. These results are obtained by a systematic use of Dobrushin's ergodic coefficient In an approach to non-linear filtering using multiplicative functionals, the pathwise filter is analyzed using the stability results for nonlinear Feynman-Kac semigroups. Provided that the signal process satisfies a mixing condition the authors obtain the asymptotic stability of the robust version of the optimal filter. In *About sur-contraction properties of Markov kernels* (Del Moral, Ledoux and Miclo), the authors use abstract relative entropy to study contraction properties of Markov transitions which generalize the classical work of Dobrushin. Of special note are the results on the logarithmic Sobolev and modified Sobolev constants based on a Dirichlet form that provide contraction estimates in an entropy distance.

In the paper *On the Applications of Maslov Optimization Theory* (Del Moral and Doisy) a rigorous theory of idempotent probability calculus is developed using the ideas on Maslov and ideas from large deviation theory. This provides an abstract setting in which to formulate the Bellman optimization principle and a class of generalized solutions for the Hamilton-Jacobi equation. This viewpoint based on probabilistic analogies has the potential to provide new tools and insights to optimal control problems.

To conclude, the main achievements of the research program of Pierre Del Moral presented in CONTRIBUTIONS A L'ESTIMATION NON LINEAIRE ET AUX SYSTEMES DE PARTICULES EN INTERACTION are:

- the identification of the central role of Feynman-Kac formulae and their stability properties to problems of non-linear filtering
- the systematic development of approximating interacting particle systems and a deep analysis of the asymptotics of the empirical measures of these systems
- new convergence results for simulated annealing and genetic algorithms for global optimization
- the development of idempotent probability with applications to optimization theory and control theory.

In each case these results are novel and yield a significant improvement on earlier approaches to the subject. In particular, Del Moral's work on interacting particle system approximations has had an important impact on the development of this approach. These demonstrate the effectiveness of this research program. Together, this body of work advances stochastic analysis and is an important contribution to nonlinear filtering and control.

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From: Donald A Dawson To: Laboratoire de Stat et Prob Date: 27/04/02 Time: 11:10:54 PM Page 5 of 5