Change Detection for Nonlinear Systems; A Particle Filtering Approach

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Abstract

In this paper we present a change detection method for nonlinear stochastic systems based on Projection Particle Filtering. The statistic for this method is chosen in such a way that it can be calculated recursively while the computational complexity of the method remains constant with respect to time. We present some simulation results that show the advantages of this method compared to linearization techniques.

1 Introduction

In many practical problems arising in quality control, fault detection, and integrity monitoring, the underlying system can be modeled as a parametric model. The parameters of such models usually can be categorized into two different sets. The first set contains the parameters that change slowly with respect to time, for example the parameters that describe the conditional density of position-velocity-orientation in a navigation system are of this type. The second set contains the parameters that are subject to sudden changes. These sudden changes are the results of a failure in the system dynamic, malfunctioning of measuring instruments, or perhaps the result of a change in the state of the system. We refer to these changes as sudden or abrupt because the time frame in which these changes happen is much smaller than the response time of the system which is limited by the nominal bandwidth of the system [1] .

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The abrupt changes in the system do not need to be catastrophic. In fact, in this paper we are interested in studying the changes that degrade the performance/accuracy/efficiency of the system, but do not stop the system from functioning. A monitoring system is responsible to detect and isolate these changes.

Online detection of abrupt changes for linear dynamical systems have been studied extensively (cf. [1] and the references therein). Unlike the linear case, change detection for nonlinear stochastic systems has not been investigated in any depth. In the cases where a nonlinear system experiences a sudden change, linearization and change detection methods for linear systems are the main tools for solving the change detection problem (see [2] for example). The reason for this lack of interest is clear; even when there is no change, the estimation of the state of the system given the observations results in an infinite dimensional nonlinear filter [3], the change in the system can only make the estimation harder.

In the last decade there has been an increasing interest in simulation based nonlinear filtering methods. These filtering methods are based on a grid-less approximation of the conditional density of the state given the observations. Grid-less simulation based filtering, now known by many different names such as Particle Filtering (PaF) [4][5], the Condensation Algorithm [6], the Sequential Monte Carlo (SMC) Method [7], and Bayesian Bootstrap Filtering [8], was first introduced in [8] and then it was rediscovered independently in [6] and [9].

The theoretical results regarding the convergence of the approximate conditional density given by PaF to the true conditional density (in some proper sense), suggests that this method is a strong alternative for nonlinear filtering [4]. The advantage of this method over the nonlinear filter is that PaF is a finite dimensional filter. The authors believe that PaF and its modifications are a starting point to study change detection for nonlinear stochastic systems. In this paper we use the results in [10] and we develop a new change detection method for nonlinear stochastic systems.

In [10] we showed that when the number of satellites is below a critical number, linearization methods such as EKF result in an unacceptable position error for an integrated INS/GPS. We also showed that the approximate nonlinear filtering methods, Projection Particle Filter (PPaF) in particular, are capable of providing an acceptable estimate of the position in the same situation.

If the carrier phase is used for position information in an integrated INS/GPS, one sudden change that happens rather often is the cycle slip. A cycle slip happens when the phase of the received signal estimated by the phase lock loop in the receiver has a sudden jump. An integrated INS/GPS with carrier phase receiver is used as an application for the method introduced in this paper. Since the proposed change detection method assumes known parameters after change, this application should not be considered a cycle slip detection method.

In Section 2 we briefly define the change detection problem. In Section 3 we review the CUSUM algorithm for linear systems with additive changes. Then in Sections 4 and 5 we present a new change detection method for nonlinear stochastic systems. In Section 6 we present some simulation results. In the last section of this paper we summarize the results and lay out the future work.

2 Change Detection: Problem Definition

On-line detection of a change can be formulated as follows [1]. Let $\mathcal{Y}_1^n = \{\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_n\}$ be a sequence of observed random variables with conditional density $p_\theta(\mathbf{y}_k|\mathbf{y}_{k-1}, \cdots, \mathbf{y}_1)$. Before the unknown change time t_0 the parameter of the conditional density, θ, is constant and equal to θ_0. After the change, this parameter is equal to θ_1. In online change detection one is interested in detecting the occurrence of such a change. The exact time and the estimation of the parameters before and after the change is not required. In case of multiple changes, we assume that the changes are detected fast enough so that in each time instance only one change has to be considered. Online change detection is performed by a stopping rule [1]

$$t_a = \inf\{n : g_n(\mathcal{Y}_1^n) \geq \lambda\}$$

where λ is a threshold, $(g_n)_{n\geq 1}$ is a family of functions, and t_a is the alarm time, i.e. the time that change is detected.

If $t_a < t_0$ then a false alarm has occurred. The criteria for choosing the parameter λ and the family of functions $(g_n)_{n\geq 1}$ is to minimize the detection delay for the fixed mean time between false alarms.

3 Additive Changes in Linear Dynamical Systems

Consider the following system:

$$\begin{array}{rclclcl} \mathbf{x}_{k+1} & = & F_k\mathbf{x}_k & + & G_k\mathbf{w}_k & + & \Gamma_k\Upsilon_{\mathbf{x}}(k,t_0) \\ \mathbf{y}_k & = & H_k\mathbf{x}_k & + & \mathbf{v}_k & + & \Xi_k\Upsilon_{\mathbf{y}}(k,t_0) \end{array} \quad (1)$$

where $\mathbf{x}_k \in \mathcal{R}^n$, $\mathbf{y}_{n\tau} \in \mathcal{R}^d$, $\mathbf{w}_k \in \mathcal{R}^q$ and $\mathbf{v}_k \in \mathcal{R}^d$ are white noise with known statistics, F_k, G_k, H_K, Γ_k, and Ξ_k are matrices of proper dimension, and $\Upsilon_{\mathbf{x}}(k,t_0)$ and $\Upsilon_{\mathbf{y}}(k,t_0)$ are the dynamic profiles of the assumed changes, of dimension $\tilde{n} \leq n$ and $\tilde{d} \leq d$, respectively. $\mathbf{w}_k$ and $\mathbf{v}_k$ are white Gaussian noise, independent of the initial condition $\mathbf{x}_0$. It is assumed that$\Upsilon_{\mathbf{x}}(k,t_0) = 0$ and $\Upsilon_{\mathbf{y}}(k,t_0) = 0$ for $k < t_0$, but we do not necessarily have the exact knowledge of the dynamic profile and the gain matrices, Γ_k and Ξ_k. The dynamic profile of change may be assumed known or unknown.

For the case of known parameters before and after change, the CUSUM [1] algorithm can be used, and it is well known that the change detection method has the following form

$$\begin{array}{rcl} t_a & = & \min\{k \geq 1 | g_k \geq \lambda\} \\ g_k & = & \max_{1\leq j\leq k} S_j^k \\ S_j^k & = & \ln \dfrac{\prod_{i=j}^k p_{\rho(i,j)}(\epsilon_i)}{\prod_{i=j}^k p_0(\epsilon_i)} \end{array} \quad (2)$$

where ϵ_i is the innovation process calculated using Kalman filtering assuming that no change occurred, and $\rho(i,j)$ is the mean of the innovation process at time j condition on the change occurred at the time i. p_0 and $p_{\rho(\cdot,\cdot)}$ are Gaussian densities with means 0, and $\rho(\cdot,\cdot)$, respectively. The covariance matrix for these two densities is the same and is calculated using Kalman filtering.

When the parameter after change is not known, the algorithm that is used for the change detection is the Generalized Likelihood Ratio (GLR) test [11]. In this case g_k is calculated as follows

$$g_k = \max_{1\leq j\leq k} \sup_{\Upsilon_{\mathbf{x}},\Upsilon_{\mathbf{y}}} S_j^k. \quad (3)$$

The solution for (3) is well known and can be found in many references [1].

Similar to nonlinear filtering, change detection for nonlinear stochastic systems results in an algorithm that is infinite dimensional. Linearization techniques, whenever applicable, are the main approximation tool for studying the change detection problem for nonlinear systems. In this setup, a nonlinear filtering problem is transformed to it linearized form through Extended Kalman Filtering (EKF) and then the same algorithms that are used for the linear Gaussian case are used for the change detection problem. Although linearization techniques are computationally efficient, they are not always applicable. In the sections to come we propose a new method based on nonlinear Particle Filtering that can be used for change detection for nonlinear stochastic systems.

4 Nonlinear Change Detection: Problem Setup

Consider the following nonlinear system

$$\begin{array}{rcl} \mathbf{x}_{k+1} & = & \mathbf{f}_k^{i_k}(\mathbf{x}_k) + G_k^{i_k}(\mathbf{x}_k)\mathbf{w}_k \\ \mathbf{y}_k & = & \mathbf{h}_k^{i_k}(\mathbf{x}_k) + \mathbf{v}_k \end{array} \quad (4)$$

where the functions $\mathbf{f}_k^{i_k}(\cdot)$ and $\mathbf{h}_k^{i_k}(\cdot)$ and the matrix $G_k^{i_k}(\cdot)$ have the proper dimensions. The noise processes $\mathbf{w}_k$,

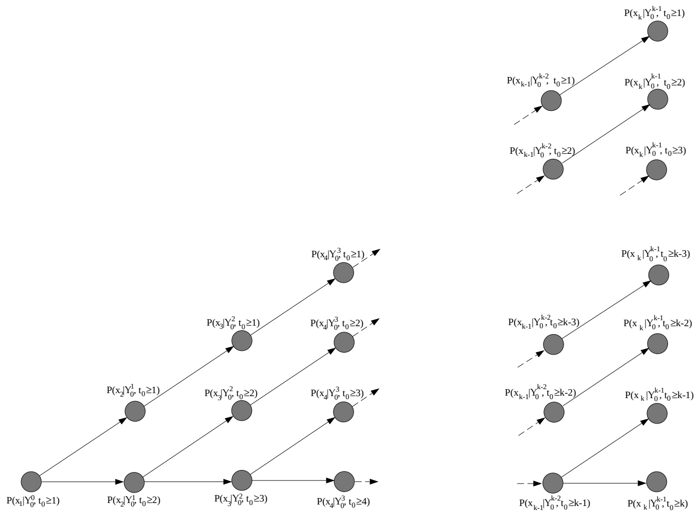

Figure 1: Combination of nonlinear filters used in the CUSUM change detection algorithm

$\mathbf{v}_k,\ k = 0, 1, \cdots$, and the initial condition $\mathbf{x}_0$ are assumed independent. We assume that

$$i_k = \begin{cases} 0 & k < t_0 \\ i & k \geq t_0, \quad i \in I \end{cases}, \tag{5}$$

where I is a countable index set. The index 0 is used for the nominal system and the system after change belongs to a countable set of systems. In this paper we assume that the set I has only one member, i.e. we assume that the parameters after the change are known.

In this setup S_j^k can be written as follows

$$S_j^k = \ln \frac{p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 = j)}{p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 > k)}. \tag{6}$$

Writing (6) in a recursive form we get

$$p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 = j) = \prod_{i=j}^{k} p(\mathbf{y}_i|\mathcal{Y}_1^{i-1}, t_0 = j) \tag{7}$$

where $p(\mathbf{y}_i|\mathcal{Y}_1^{i-1}, t_0 = j)$ can be written as follows

$$p(\mathbf{y}_i|\mathcal{Y}_1^{i-1}, t_0 = j) = \int_{\mathbf{x}_i} p(\mathbf{y}_i|\mathbf{x}_i) p(\mathbf{x}_i|\mathcal{Y}_1^{i-1}, t_0 = j) d\mathbf{x}_i. \tag{8}$$

To find $p(\mathbf{x}_i|\mathcal{Y}_1^{i-1}, t_0 = j)$ in (8), one needs to find an approximation for the corresponding nonlinear filter. We assume that this approximation is done using either Particle Filtering (PaF) or Projection Particle Filtering (PPaF) [10].

To calculate the likelihood ratio in (6), we must calculate the conditional densities of the state given the observation for two hypothesis (changed occurred at j and change occurred after k change). This means that two nonlinear filters should be implemented just to compare these two hypothesis. Therefore, it is clear that to use an algorithm similar to (2), k parallel nonlinear filter should be implemented.

In Figure 1, we see that the computational complexity of the CUSUM algorithm grows linearly with respect to time. In most applications this growth is not desirable. One possible way to approximate the CUSUM algorithm is to truncate the branches that are forked from the main branch in Figure 1. We will explain this truncation procedure and its technical difficulties in the next few lines.

Recall that the main branch (horizontal) and the branches forked from it in Figure 1 are representing a series of nonlinear filters with specific assumptions on the change time. The dynamic and the observation equation for all forked branches are the same and the only difference is the initial density. If the conditional density of the state given the observation for a nonlinear system with the wrong initial density converges (in some meaningful way) to the true conditional density (initialized by the true initial density), we say that the corresponding nonlinear filter is **Asymptotically Stable** [12].

For asymptotically stable nonlinear filters the forked branches in Figure 1 converge to a single branch, therefore there is no need to implement several parallel nonlinear filters. In other words, after each branching the independent nonlinear filter is used for a period of time and then this branch converges to the the branches that have forked earlier, i.e. joins them. The time needed for the branch of the independent nonlinear filter before joining the other forked branches depends on the convergence rate and the target accuracy of the approximation.

Although the procedure mentioned above can be used for asymptotically stable nonlinear filters, there are several problems associated to this method. The known theoretical results for identifying asymptotically stable filters is limited to either requiring ergodicity and the compactness of the state space [13] [14] [15] or very special cases of the observation equation [12]. The rate of convergence of the filters in different branches is another potential shortcoming of the mentioned procedure. If the convergence rate is low in comparison with the the rate of parameter change in the system, then the algorithm cannot take advantage of this convergence.

5 Nonlinear Change Detection: Non Growing Computational Complexity

In this section we introduce a new statistic to overcome the problem of growing computational complexity for the change detection method. We show that this statistic can be calculated recursively.

Consider the following statistic

$$T_j^k = \ln \frac{p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 \in \{j, \cdots, k\})}{p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 > k)}. \tag{9}$$

For the rest of this paper we assume that conditioned on change, the change time, t_0, is distributed uniformly, i.e.

$$P(t_0 = i|t_0 \in \{j, \cdots, k\}) = \begin{cases} \frac{1}{k-j+1} & i \in \{j, \cdots, k\} \\ 0 & \text{otherwise} \end{cases}. \tag{10}$$

With this assumption we have

$$
\begin{aligned}
&p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 \in \{j, \cdots, k\}) \\
&\quad = p(\mathcal{Y}_j^k, t_0 \in \{j, \cdots, k\}|\mathcal{Y}_1^{j-1}, t_0 \in \{j, \cdots, k\}) \\
&\quad = p(\mathcal{Y}_j^k, t_0 = j \quad |\mathcal{Y}_1^{j-1}, t_0 \in \{j, \cdots, k\}) + \\
&\qquad p(\mathcal{Y}_j^k, t_0 = j+1|\mathcal{Y}_1^{j-1}, t_0 \in \{j, \cdots, k\}) + \\
&\qquad \vdots \\
&\qquad p(\mathcal{Y}_j^k, t_0 = k \quad |\mathcal{Y}_1^{j-1}, t_0 \in \{j, \cdots, k\}) \\
&\quad = \tfrac{1}{k-j+1} \big(p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 = j) + \\
&\qquad\quad p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 = j+1) + \\
&\qquad\quad \cdots + p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 = k) \big) \,,
\end{aligned}
$$

therefore,

$$
\begin{aligned}
T_j^k &= \ln \frac{p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 \in \{j, \cdots, k\})}{p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 > k)} \\
&= \ln \left(\frac{1}{k-j+1} \sum_{i=j}^{k} \frac{p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 = i)}{p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 > k)} \right).
\end{aligned}
$$

In other words T_j^k can be written as follows

$$
T_j^k = \ln \left(\frac{1}{k-j+1} \sum_{i=j}^{k} \exp(S_i^k) \right). \tag{11}
$$

If we define $\hat{S}_j^k = \frac{p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 = j)}{p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 > k)}$ then

$$
\hat{T}_j^k = \frac{1}{k-j+1} \sum_{i=j}^{k} \hat{S}_i^k, \tag{12}
$$

where $\hat{T}_j^k = \exp(T_j^k)$.

The change detection algorithm based on statistic T_j^k can be presented as follows

$$
t_a = \min\{k \geq j \mid T_j^k \geq \lambda \;\; \text{or} \quad T_j^k \leq -\alpha\}, \tag{13}
$$

where j is the last time that $g_k \geq \lambda$ or $g_k \leq -\alpha$, and $\lambda > 0$ and $\alpha > 0$ are chosen such that the detection delay is minimum for a fixed mean time between two false alarms. Using (2) and (12), we try to find a relation between the detection algorithm (13) and the CUSUM algorithm. Assume two possible extreme cases. The first one is the case where $S_i^k = c, \forall i \in \{j, \cdots, k\}$. In this case it is clear that $T_j^k = S_i^k \quad \forall i \in \{j, \cdots, k\}$, and therefore, the performance of the two methods with the same thresholds is the same. In the second case we assume that $\exists i, l \in \{j, \cdots, k\}$ such that $S_i^k >> S_l^k, \;\; l \neq i$. Therefore, it can be seen that $T_j^k \simeq S_l^k - \ln(k-j+1)$, i.e. T_j^k is degraded by $-\ln(k-j+1)$. With this simple analysis we can conclude that

$$
\max_{i \in \{j, \cdots, k\}} S_i^k - \ln(k-j+1) \leq T_j^k \leq \max_{i \in \{j, \cdots, k\}} S_i^k. \tag{14}
$$

Therefore, with the same thresholds for both detection methods, (14) can be used to find the bounds for the performance of the detection algorithm in (13) with respect to the CUSUM algorithm. We emphasize that the thresholds used for detection method (13) need not be the same as the thresholds in the CUSUM algorithm, in fact, they should be optimum according to the criteria for the mean detection delay for the detection method in (13).

The main advantage of using the statistic T_j^k over S_j^k is the fact that T_j^k can be calculated recursively without growth in the computational complexity of the method with respect to time. We can rewrite $p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 \in \{j, \cdots, k\})$ as follows

$$
p(\mathcal{Y}_j^k|\mathcal{Y}_1^{j-1}, t_0 \in \{j, \cdots, k\}) = \prod_{i=j}^{k} p(\mathbf{y}_i|\mathcal{Y}_1^{i-1}, t_0 \in \{j, \cdots, k\}).
$$

Using (10) we have

$$
\begin{aligned}
&p(\mathbf{y}_i|\mathcal{Y}_1^{i-1}, t_0 \in \{j, \cdots, k\}) \\
&\quad = p(\mathbf{y}_i, t_0 \in \{j, \cdots, k\}|\mathcal{Y}_1^{i-1}, t_0 \in \{j, \cdots, k\}) \\
&\quad = p(\mathbf{y}_i, t_0 \in \{j, \cdots, i\}|\mathcal{Y}_1^{i-1}, t_0 \in \{j, \cdots, k\}) \;+ \\
&\qquad p(\mathbf{y}_i, t_0 > i|\mathcal{Y}_1^{i-1}, t_0 \in \{j, \cdots, k\}) \\
&\quad = \tfrac{i-j+1}{k-j+1} p(\mathbf{y}_i|\mathcal{Y}_1^{i-1}, t_0 \in \{j, \cdots, i\}) \;+ \\
&\qquad \tfrac{k-i}{k-j+1} p(\mathbf{y}_i|\mathcal{Y}_1^{i-1}, t_0 > i)
\end{aligned} \tag{15}
$$

From (15) it is clear that we need only to calculate two types of functions. These two functions are $p(\mathbf{y}_i|\mathcal{Y}_1^{i-1}, t_0 \in \{j, \cdots, i\})$ and $p(\mathbf{y}_i|\mathcal{Y}_1^{i-1}, t_0 > i)$. To calculate these two functions we can use the following

$$
\begin{aligned}
&p(\mathbf{y}_i|\mathcal{Y}_1^{i-1}, t_0 \in \{j, \cdots, i\}) \\
&= \int p(\mathbf{y}_i|\mathbf{x}_i, \; t_0 \in \{j, \cdots, i\}) \\
&\qquad\qquad p(\mathbf{x}_i|\mathcal{Y}_1^{i-1}, t_0 \in \{j, \cdots, i\}) d\mathbf{x}_i \\
&= \tfrac{1}{i-j+1} \int p_1(\mathbf{y}_i|\mathbf{x}_i) p(\mathbf{x}_i|\mathcal{Y}_1^{i-1}, \; t_0 = i) d\mathbf{x}_i \;+ \\
&\tfrac{i-j}{i-j+1} \int p_1(\mathbf{y}_i|\mathbf{x}_i) p(\mathbf{x}_i|\mathcal{Y}_1^{i-1}, \; t_0 \in \{j, \cdots, i-1\}) d\mathbf{x}_i \\
&= \tfrac{1}{i-j+1} \int p_1(\mathbf{y}_i|\mathbf{x}_i) p(\mathbf{x}_i|\mathcal{Y}_1^{i-1}, \; t_0 > i-1) d\mathbf{x}_i \;+ \\
&\tfrac{i-j}{i-j+1} \int p_1(\mathbf{y}_i|\mathbf{x}_i) p(\mathbf{x}_i|\mathcal{Y}_1^{i-1}, \; t_0 \in \{j, \cdots, i-1\}) d\mathbf{x}_i
\end{aligned} \tag{16}
$$

and

$$
\begin{aligned}
&p(\mathbf{y}_i|\mathcal{Y}_1^{i-1}, t_0 > i) \\
&\quad = \int p(\mathbf{y}_i|\mathbf{x}_i, \; t_0 > i) p(\mathbf{x}_i|\mathcal{Y}_1^{i-1}, \; t_0 > i) d\mathbf{x}_i \\
&\quad = \int p_0(\mathbf{y}_i|\mathbf{x}_i) p(\mathbf{x}_i|\mathcal{Y}_1^{i-1}, t_0 > i-1) d\mathbf{x}_i \;,
\end{aligned} \tag{17}
$$

where $p_0(\mathbf{y}_i|\mathbf{x}_i)$ and $p_1(\mathbf{y}_i|\mathbf{x}_i)$ are the conditional densities of the observation given the state of the system before and after the change, respectively. To calculate these two functions two conditional densities, $p(\mathbf{x}_i|\mathcal{Y}_1^{i-1}, t_0 > i-1)$ and $p(\mathbf{x}_i|\mathcal{Y}_1^{i-1}, t_0 \in \{j, \cdots, i-1\})$ should be found. These two conditional densities can be calculated recursively as follows

$$
\begin{aligned}
&p(\mathbf{x}_i|\mathcal{Y}_1^{i-1}, t_0 > i-1) \\
&\quad = \int p(\mathbf{x}_i|\mathbf{x}_{i-1}, \; t_0 > i-1) \\
&\qquad\qquad p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-1}, \; t_0 > i-1) d\mathbf{x}_{i-1} \\
&\quad = \int p_0(\mathbf{x}_i|\mathbf{x}_{i-1}) p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-1}, \; t_0 > i-1) d\mathbf{x}_{i-1} \;,
\end{aligned} \tag{18}
$$

where $p_0(\mathbf{x}_i|\mathbf{x}_{i-1})$ is the conditional density of the state at time i given the state at time $i-1$ assuming that no change has happened up until time $i-1$. The recursion is complete

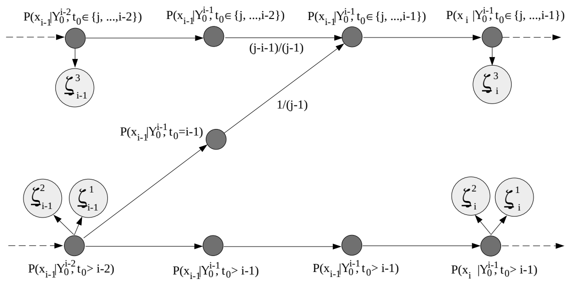

Figure 2: Implementation of the nonlinear filters used in the change detection algorithm in (13)

with

$$
\begin{aligned}
&p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-1},\ t_0>i-1)\\
&=\frac{p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-2},\ t_0>i-1)p(\mathbf{y}_{i-1}|\mathbf{x}_{i-1},\ t_0>i-1)}{\int p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-2},\ t_0>i-1)p(\mathbf{y}_{i-1}|\mathbf{x}_{i-1},\ t_0>i-1)d\mathbf{x}_{i-1}} \qquad (19)\\
&=\frac{p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-2},\ t_0>i-2)p_0(\mathbf{y}_{i-1}|\mathbf{x}_{i-1})}{\int p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-2},\ t_0>i-2)p_0(\mathbf{y}_{i-1}|\mathbf{x}_{i-1})d\mathbf{x}_{i-1}},
\end{aligned}
$$

and it is assumed that the initial density of the state is known. (18) and (19) are in fact the equations for the nonlinear filter assuming that no change has happened. For the other conditional density we have

$$
\begin{aligned}
&p(\mathbf{x}_i|\mathcal{Y}_1^{i-1},\ t_0\in\{j,\cdots,i-1\})\\
&=\int p(\mathbf{x}_i|\mathbf{x}_{i-1},\ t_0\in\{j,\cdots,i-1\})\\
&\qquad p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-1},\ t_0\in\{j,\cdots,i-1\})d\mathbf{x}_{i-1} \qquad (20)\\
&=\tfrac{1}{j-i}\int p_1(\mathbf{x}_i|\mathbf{x}_{i-1})p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-1},\ t_0=i-1)d\mathbf{x}_{i-1}\\
&\quad+\tfrac{j-i-1}{j-i}\int p_1(\mathbf{x}_i|\mathbf{x}_{i-1})\\
&\qquad p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-1},\ t_0\in\{j,\cdots,i-2\})d\mathbf{x}_{i-1},
\end{aligned}
$$

where $p_1(\mathbf{x}_i|\mathbf{x}_{i-1})$ is the conditional density of the state at time i given the state at time $i-1$ assuming that a change has occurred. To complete the recursion formula we have

$$
\begin{aligned}
&p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-1},\ t_0=i-1)\\
&=\frac{p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-2},\ t_0=i-1)p(\mathbf{y}_{i-1}|\mathbf{x}_{i-1},\ t_0=i-1)}{\int p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-2},\ t_0=i-1)p(\mathbf{y}_{i-1}|\mathbf{x}_{i-1},\ t_0=i-1)d\mathbf{x}_{i-1}} \qquad (21)\\
&=\frac{p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-2},\ t_0>i-2)p_1(\mathbf{y}_{i-1}|\mathbf{x}_{i-1})}{\int p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-2},\ t_0>i-2)p_1(\mathbf{y}_{i-1}|\mathbf{x}_{i-1})d\mathbf{x}_{i-1}},
\end{aligned}
$$

and

$$
\begin{aligned}
&p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-1},\ t_0\in\{j,\cdots,i-2\})=\\
&\frac{p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-2},\ t_0\in\{j,\cdots,i-2\})p(\mathbf{y}_{i-1}|\mathbf{x}_{i-1},\ t_0\in\{j,\cdots,i-2\})}{\int p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-2},\ t_0\in\{j,\cdots,i-2\})p(\mathbf{y}_{i-1}|\mathbf{x}_{i-1},\ t_0\in\{j,\cdots,i-2\})d\mathbf{x}_{i-1}} \qquad (22)\\
&=\frac{p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-2},\ t_0\in\{j,\cdots,i-2\})p_1(\mathbf{y}_{i-1}|\mathbf{x}_{i-1})}{\int p(\mathbf{x}_{i-1}|\mathcal{Y}_1^{i-2},\ t_0\in\{j,\cdots,i-2\})p_1(\mathbf{y}_{i-1}|\mathbf{x}_{i-1})d\mathbf{x}_{i-1}}.
\end{aligned}
$$

Figure 2 shows the implementation of equations (18) through (22), it can be seen that the complexity of the implemented nonlinear filter does not grow with time. Using Figure 2 and the definition of T_j^k we have

$$
T_j^k=\sum_{i=j}^{k}\ln\left(\frac{1}{k-j+1}\left((k-i)+\frac{\varsigma_i^2}{\varsigma_i^1}+(i-j)\frac{\varsigma_i^3}{\varsigma_i^1}\right)\right), \quad (23)
$$

where

$$
\begin{aligned}
\varsigma_i^1 &= p(\mathbf{y}_i|\mathcal{Y}_1^{i-1},\ t_0>i)\\
\varsigma_i^2 &= p(\mathbf{y}_i|\mathcal{Y}_1^{i-1},\ t_0=i)\\
\varsigma_i^3 &= p(\mathbf{y}_i|\mathcal{Y}_1^{i-1},\ t_0\in\{j,\cdots,i-1\}).
\end{aligned}
$$

6 Simulations and Results

In [10] we showed that for an integrated INS/GPS when the number of satellites is less than a critical number, Projection Particle Filtering (PaF) provides a very accurate estimate of the position while the position solution given by EKF is unacceptable. In this paper we use the same example to apply the change detection method in (13). Similar to [10] for a critical situation (low number of observable satellites) linearization does not work. On the other hand the CUSUM algorithm leads to a growth in computational complexity with respect to time, therefore, at this point a natural selection for a change detection algorithm is the method in (13). We wish to emphasize that in the example given in this section we assume that the parameter of change before and after change is known and the only unknown parameter is the change time. In future we will address the more general problem of unknown change parameters.

The dynamics of an integrated INS/GPS is given in [10]. The observation given by a Differential GPS is very similar to the one given in the same reference. The only difference is that we assume that the signal associated to one of the satellites experiences an abrupt change, i.e. we assume a known cycle slip in one of the channels. We invite the reader to see [10] for details.

For this simulation we simply chose an 11 dimensional Gaussian density for the projection PaF. This choice of density makes the random vector generation easy and computationally affordable. To be able to use the projection PaF, we used maximum likelihood to estimate the parameters of the Guassian density before and after Bayes' correction.

In this simulation, we used two Novatel GPS receivers to collect the navigation data on April 2, 2000. From the collected data, we extracted the position information of the satellites, the pseudo range, and the carrier phase measurement noise powers for the L1 frequency. Using the collected information we generated the pseudo range and the carrier phase data for one static and one moving receiver (base and rover, respectively). Here we assume for the carrier phase measurement the integer ambiguity problem is already solved. We also assumed that the phase lock loop associated to satellite one experiences a cycle slip and the phase changes suddenly. The size of the change is assumed to be one cycle. The movement of the INS/GPS platform was simulation based and the measurement data measured by the accelerometers, the gyros, the GPS pseudo range,

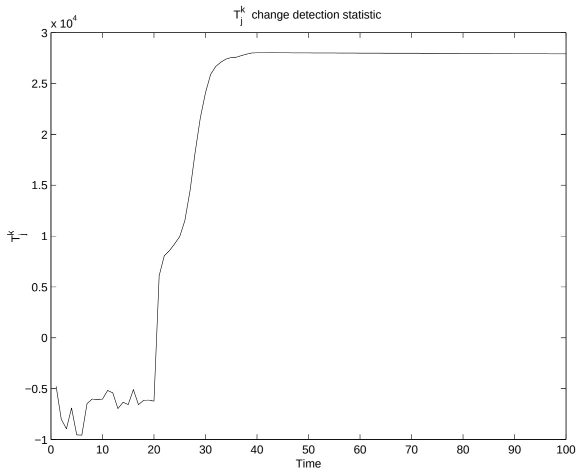

Figure 3: This figure shows the plot of T_j^k with respect to time. At time $t = 15$, the receiver looses 3 satellites. We assume that the cycle slip in channel one occurred at time $t = 20$.

and the GPS carrier phase data were generated according to that movement.

In the simulation we assumed that the GPS receiver starts with 6 satellites. At time $t = 15$, the receiver looses 3 satellites. We assume that the cycle slip in channel one occurred at time $t = 20$. In Figure 3 we have plotted T_j^k with respect to time. From the figure it is clear that the change time is estimated accurately.

7 Conclusion and Future Work

In this paper we developed a new method for the detection of abrupt changes for known parameters after change. We showed that unlike the CUSUM algorithm, the statistic in this method can be calculated recursively for nonlinear stochastic systems. In future, we intend to extend our results to the case where the parameters after change are unknown. The major obstacle in this extension is the complexity of the change detection method. Another subject that requires further investigations is the comparison of this method with other existing methods.

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