On housing booms and credit market conditions: A state space model*

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Abstract: The massive and persistent increases of house prices in the US before the 2008/09 financial crisis show the limit of asset pricing models which presume a fixed steady state. In this paper, we propose a state space model characterized by a time varying long-term state of the log house price to rent ratio. An increasing trend in the long-term price to rent ratio from early 1990s to 2007 is identified in the data. These increases were significantly influenced by decreasing real mortgage rates and the increased securitization of residential mortgage loans, especially since 2002. The impulse response analysis from the estimated model shows that the effect of securitization activities on the long-run price to rent ratio is three times larger than the one of real mortgage rates.

Keywords: House prices, real interest rates, securitization, state space models, particle filter.
JEL Classification: E4, G1

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1 Introduction

Prior to the 2008/2009 financial crisis, the house price in the US had increased to an unprecedent level. In real terms the FHFA house price index increased by 52% from 1991 to 2006. This persistent increase of the house price has lead to lively discussions in the literature about its origin. Among other factors such as the market psychology (Shiller; 2007), easy credit market conditions, as, for instance, low real mortgage rates and fostered securitization activities of residential mortgage loans, may have contributed to the increasing house price.

Although there are theoretical arguments outlining the effect of real mortgage rates and securitization on the house price, the empirical macroeconomic evidence is not yet conclusive. Lower real mortgage rates imply lower financial costs of the mortgage loan, which may lead to an increasing demand for housing and an associate acceleration of prices. Nevertheless, the empirically diagnosed effects of real rates on the housing prices are either weak or not significant (Mankiw and Weil; 1989; Muellbauer and Murphy; 1997; Himmelberg, Mayer and Sinai; 2005; Brunnermeier and Julliard; 2008; Glaeser, Gottlieb and Gyourko; 2010). With regard to securitization activities, additional funding sources for mortgage loans through securitization endogenize the credit supply (Shin; 2009). Mian and Sufi (2009) show that the expansion in subprime mortgage credit from 2002 to 2005 was closely correlated with the increasing securitization of subprime mortgages using ZIP code-level data. The evidence from loan level data also supports the view that the underwriting standards of mortgage loans were significantly relaxed for the period of 2001 to 2006 (Keys, Mukherjee, Seru and Vig; 2010). Despite these findings, at the level of macroeconomic aggregation, no clear supporting evidence for the effect of weakened underwriting standards on housing prices has yet been found (Brunnermeier and Julliard; 2008; Glaeser, Gottlieb and Gyourko; 2010).

Apart from the lack of macroeconomic evidence for the effect of credit market conditions on house markets, there is a further challenge to the asset market approach to valuing housing prices. When regarding a house as an asset, the current price to rent ratio can be approximated by discounting expected future rent growth rates and returns on housing, as considered by Brunnermeier and Julliard (2008). This approach parallels
the modeling of the stock price to dividend ratio proposed by Campbell and Shiller (1989), and relies on a linear approximation around the constant mean of the price to rent ratio. However, empirical observations shed doubt on the suitability of such a model of the price to rent ratio. Although the price to rent ratio is bounded in a certain range, it shows characteristics of a non-stationary process, where shocks have persistent effects over a long period. Consequently, the linear approximation based on the constant mean of the price to rent ratio leads to approximation errors with a clear trend. A-priori, such evidence is better in line with the notion of a time-varying long-term state rather than one with a fixed steady state of the price to rent ratio. Intuition from the Gordon growth model implies that changes in the long-term state of the price to rent ratio can be due to persistent changes in expectations of either future rent growth rates or returns on houses.

This paper proposes a state space model with a latent financial state variable reflecting the long-term state of the Price to Rent ratio, abbreviated as PtR henceforth. The observation equation decomposes the log PtR into three components: A deterministic term, discounted expected future returns on houses and rent growth rates, and an error measure. The state variable appears in the first two components in a non-linear fashion. We estimate the non-linear state space model by means of particle filter techniques. The estimation results show that a time-varying state is supported by quarterly US data from 1982 to 2009. The model for the log PtR with the time-varying state outperforms its counterpart with the constant state variable according to model implied log-likelihood statistics. In addition, the model with constant state variable results in overly large error (mispricing) terms for the period from 2004 to 2009. It cannot explain the log PtR in time periods with persistent price changes.

After the extraction of a time varying financial state, we investigate the response of the long-term state of the log PtR to credit market conditions. We apply Vector Error Correction Models (VECMs) to analyse relationships among the estimated financial state from the house market, real mortgage rates, and securitization activities. All three variables are treated as endogenous variables in the model. Using quarterly US data from 1991 to 2009, we show that both decreasing real mortgage rates and increasing securitization activities contributed significantly to increases in the financial state, and thus, increases in the long-term PtR. Accord-
ing to recursive estimates of error correction coefficients, the long-term state of the log PtR has adjusted to credit conditions significantly and with an increasing speed since 2002.

The identified linkage can be interpreted as a house market equation. While the financial state from the house market reacted to changes in real mortgage rates and securitization activities, real mortgage rates didn’t respond to changes in the financial state or securitization. Recursive estimates of the adjustment coefficients show that the real mortgage rate is weakly exogenous to its cointegration relation with the other variables. Similar to the real mortgage rate, overall securitization activities are also diagnosed weakly exogenous.

Moreover, securitization played the most important role in describing the recent accelerations of the long-term PtR. The impact of a standardized shock in securitization on the financial state is 3 times larger than the impact of a standardized shock in the real mortgage rates, and 63 times larger than a standardized shock in the state variable itself. In addition, an isolated shock in the financial state does not have a persistent effect on itself. This evidence might be consistent with the view that changes in the mass psychology of the home buyer alone (which corresponds to shocks in the state of PtR) are not sufficient to explain persistent increases of house prices.

This paper can be related to a growing body of macroeconomic literature which use persistent changes in fundamentals to explain the striking changes of stock market valuation ratios that took place in the 1990s. These models address the persistent declines in expected returns (for example, Vissing-Jorgensen; 2002; Calvet and Gonzalez-Eiras; 2004; McGrattan and Prescott; 2005; Guvenen; 2009; Lettau, Ludvigson and Wachter; 2008; Lustig and Van Nieuwerburgh; 2010), and the increase in the steady-state growth rate of the economy (e.g. Jermann and Quadrini; 2007). As pointed out by Lettau and Van Nieuwerburgh (2008), either of these changes lead to a persistent decline in the mean of the stock price to dividend ratio. By allowing shifts in the steady state, Lettau and Van Nieuwerburgh (2008) found that the price to dividend ratio robustly forecasts returns in their sample. In light of these developments in the literature about the stock price to dividend ratio, this paper investigates changes in the steady state of the house price relative to rent. A particular novelty lies in the formalization and estimation of smooth
changes in the long-term state of the house price to rent ratio through the non-linear dynamic asset pricing model, and relating these changes to real mortgage rates and securitization.

It is worth mentioning that focusing on the house price relative to the rent allows us to rule out symmetric movements due to changes in fundamentals such as construction costs, income, the unemployment rate or demographics. Thus, the effect of the credit markets can be brought out in the spot light. Even when there are other factors which may affect house markets and rental markets in an asymmetric manner (such as the pride of being home owners), dynamics of the house price relative to rent are not influenced by such factors as long as they are relatively constant over time.

The next section documents the two empirical observations for the log PtR in the US which motivate modeling the log PtR by means of a flexible financial state process. Section 3 describes the state space model of the log PtR in detail. The model is quantified in Section 4. Section 5 specifies the subset VECMs. The evidence is discussed in Section 6. Section 7 concludes. Detailed descriptions of the data are provided in the Appendix.

2 Empirical observations

This section documents the two empirical observations, the persistency in the log PtR and it’s approximation error that is obtained from a model expansion around an implied time invariant steady state. We use quarterly data of the FHFA housing price index and the rent of primary residence as a component of the CPI for the time period of 1975 to 2009. To obtain the log PtR illustrated in Figure 1, the housing price index is scaled so that the mean of the log PtR is about 4.1581, as reported by Ayuso and Restoy (2006) for a similar time period.

The log PtR seems to be a limited, but persistent process. The boundedness of the log PtR can be easily observed from Figure 1. When the housing price has been built up excessively high relative to the rent, the ‘bubble’ will eventually burst. Thus, there is an upper bound of the log PtR. Notably, the same argument applies to the existence of a lower bound. However, being bounded in the certain range, the log PtR shows
characteristics of a non-stationary variable: shocks have persistent effects on the process. This can be seen from the persistent increases in the log PtR from 2000 to 2005.

A bounded non-stationary process, as considered by Cavaliere and Xu (2011), can reconcile these two phenomena. Cavaliere and Xu (2011) consider a class of time series processes which are non-stationary and simultaneously have bounds either from below or/and above. The bounds can be due to construction or policy controls. Obvious examples are budget shares, unemployment rates, nominal interest rates, or target zone exchange rates. This paper extends this concept to a persistent process which is bounded ultimately by the fundamentals. To test the persistence (non-stationarity) for bounded processes, Cavaliere and Xu (2011) proposed a new class of unit root tests taking the bounds into account. For conventional unit root tests, the null hypothesis is the unbounded random walk. For the new tests proposed by Cavaliere and Xu (2011), the null hypothesis is the bounded random walk and the alternative hypothesis is the bounded stationary process. They show that the critical values of the unit root test statistic become smaller when the bounds are taken into account. As a result, if the null hypothesis of the unit root process without bound is not rejected, then the null hypothesis of the unit root process with bound is even more unlikely to be rejected for the same process.

The bounded non-stationarity of the log PtR is confirmed by the unit root diagnostics. Table 1 reports the results from conventional unit root tests. Considering the log PtR from 1975 to 2009, all unit root statistics do not obtain a rejection of the unit root hypothesis at the 10% significance level. Although we do not know the latent bounds of the log PtR, as mentioned before taking bounds into account results in a smaller critical value, and thus a lower likelihood to reject the non-stationarity. Therefore, the log PtR can be reasonably approximated in terms of a bounded non-stationary process. For such a process, its mean does not converge to a fixed value although it is limited within a certain range.

As the next step, following Campbell and Shiller (1989), we investigate the approximation errors from the present value model of the log PtR when a constant mean is considered. Let $P_t$ and $L_t$ denote the observed price and rental payment of housing at the end of period $t$. The
realized log gross return at the end of period $t + 1$ is

$$r_{t+1} \equiv \ln(P_{t+1} + L_{t+1}) - \ln(P_t)$$

$$= -\eta_t + \ln(\exp(\eta_{t+1}) + 1) + \Delta l_{t+1},$$

(1)

where $\eta_t = \ln(P_t) - \ln(L_t)$ is the log PtR and $\Delta$ is the difference operator such that e.g. $\Delta l_t = l_t - l_{t-1}$. Lower case letters refer to the natural log of the corresponding upper case letters. Equation (1) is nonlinear in terms of $\eta_{t+1}$. To obtain a linear approximation, a first-order Taylor expansion of (1) is commonly applied. Around the fixed point $\eta$, the linear approximation reads as

$$r_{t+1} \simeq \kappa - \eta_t + \rho \eta_{t+1} + \Delta l_{t+1},$$

(2)

with $\rho \equiv \frac{1}{1 + \exp(-\eta)}$ and $\kappa \equiv -\ln(\rho) - (1 - \rho) \ln(1/\rho - 1)$. Equation (2) can be thought as a formalization of the current log PtR through the future log PtR, return and rent growth rate. Notably, $\rho = \frac{P}{P+R}$ reflects the importance of the price relative to the sum of the price and the rent. The higher the price relative to the rent, the more weight is attached to the future log PtR in the pricing equation.

Equation (2) is the corner stone of the present value model for the PtR. The approximation parameter $\rho$ is a function of the considered fixed steady state $\eta$. Up to date, the mean value of the log PtR in the observed sample is used as the fixed point (Brunnermeier and Julliard; 2008) to obtain the approximation. This follows the idea that presuming stationarity of the PtR, the first-order Taylor expansion around the steady state provides the best linear approximation on average. Iterating equation (2) forward obtains

$$\eta_t \simeq \frac{\kappa}{1 - \rho} + \sum_{i=1}^{\infty} \rho^{i-1}(\Delta l_{t+i} - r_{t+i}) + \lim_{i \to \infty} \rho^i \eta_{t+i}.$$  

(3)

Equation (3) provides a linear approximation of the current log PtR ($\eta_t$) around its constant mean ($\eta$).

We evaluate the approximation error by comparing the log PtR with the right hand side of the equation (3), where the terminal value of $\eta_T$ is set to the last observation from the sample, i.e. 2009:Q2. The approximation error for the period of 1990:Q1 to 2004:Q2 is illustrated
in Figure 2. The approximation error with a constant mean shows a clear upward sloping and persistent trend. It appears stationary around a trend, but non-stationary around a constant mean. Common unit root tests confirm the nonstationarity.

Summarizing these two empirical features highlights that the present value model with a fixed steady state may not be fully appropriate to study the dynamics of the log PtR, comprising persistent components.

3 The state space model

In this section, we propose a state space model incorporating a time-varying long-term state of the log PtR. Assume that the long-term state of \( \eta_t \) can be time-varying, and denote it by \( \overline{\eta}_t \). A linear approximation for equation (1) can be obtained around \( \overline{\eta}_t \) as

\[
\begin{align*}
    r_{t+1} &\simeq \kappa_t - \eta_t + \rho_t \eta_{t+1} + \Delta l_{t+1}, \\
    \text{with } \rho_t &\equiv \frac{1}{1 + \exp(-\overline{\eta}_t)} \\
    \text{and } \kappa_t &\equiv -\ln(\rho_t) - (1 - \rho_t) \ln(1/\rho_t - 1).
\end{align*}
\]

This equation relates current \( \eta_t \) to future \( \eta_{t+1}, r_{t+1}, \) and \( \Delta l_{t+1} \). A time varying \( \rho_t \) implies a time-varying weight attached to the future cash flow.

To obtain an explicit form of the iterated version of equation (4), which is comparable to equation (3), we make the following assumptions: (i) \( \rho_t \) is a martingale processes, \( E_t(\rho_{t+i}) = \rho_t \); (ii) \( \kappa_t \) is a martingale processes, \( E_t(\kappa_{t+i}) = \kappa_t \); (iii) \( E_t(\rho_t \eta_{t+i+1}) = E_t(\rho_{t+i}) E_t(\eta_{t+i+1}) \), with \( i \geq 1 \). Assumption (i) is consistent with the intuition that current situations influence the agents’ expectation about the future. As the survey from Case and Shiller (quoted for example in Shiller (2007)) shows, times and places with high current home prices show high expectations of future home prices. It seems also sensible to believe that housing market participants update their belief of \( \rho_t \) with the advent of new information after each period. At the first glance, Assumption (ii) seems to be not in line with Assumption (i). Given that \( \rho_t \) is a martingale process and \( \kappa_t \) is a concave function of \( \rho_t, \kappa_t \) is a submartingale process, i.e. \( E_t(\kappa_{t+i}) \geq \kappa_t \). This can be shown by the Jensen’s inequality. However, the approximation error due to the martingale assumption of \( \kappa_t \) in
Assumption (ii) is very small. The degree of the concaveness in $\kappa(\rho_t)$ for the sensible range $[0.98, 0.99]$ of $\rho_t$ for quarterly data is negligible. For any $b \in [0, 1]$ and any $\rho_1, \rho_2 \in [0.98, 0.99]$, the maximal difference between $[b\kappa(\rho_1) + (1 - b)\kappa(\rho_2)]$ and $\kappa(b\rho_1 + (1 - b)\rho_2)$ is about 0.00086. Even when a wider range of $\rho_t$, for instance $[0.90, 0.99]$, is considered, the maximal difference between the two is still as small as 0.02. Finally, Assumption (iii) implies that the conditional expectation of log PtR at period $t + 1$ is uncorrelated to the conditional expectation of long term state of log PtR at period $t$.

Given these assumptions, taking the conditional expectation and iterating equation (4) forward yields

$$\eta_t \simeq \frac{\kappa_t}{1 - \rho_t} + \sum_{i=1}^{\infty} \rho_t^{i-1} E_t(\Delta l_{t+i} - r_{t+i}) + \lim_{i \to \infty} \rho_t^{i} E_t \eta_{t+i}. \quad (5)$$

This equation approximates the log PtR by a deterministic term, discounted expected future rent growth rates and returns, and the discounted terminal value of the log PtR. Compared to equation (3), the present value model in (5) allows for a time-varying deterministic term, which is a function of the long-term state of log PtR. Since the long-term state of log PtR is time-varying, the future cash flows are also discounted at a time-varying rate $\rho_t$. The consideration of a time-varying state of log PtR is similar to Lettau and Van Nieuwerburgh (2008). However, different as Lettau and Van Nieuwerburgh (2008), this log linear framework does not explicitly formulate the long-term state of $\Delta l_{t+i}$ or $r_{t+i}$. The concentration on the time-varying state of log PtR allows us to estimate the latent state through a state space model.

The observation equation based on equation (5) in the state space model is

$$\eta_t = \frac{\kappa_t}{1 - \rho_t} + \sum_{i=1}^{\infty} \rho_t^{i-1} \tilde{E}_t(\Delta l^e_{t+i} - r^e_{t+i}) + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2_t), \quad (6)$$

with $t = t_0, t_0 + 1, \ldots, T$, and the error term $\varepsilon_t$ can capture rational bubbles ($\lim_{i \to \infty} \rho_t^{i} E_t \eta_{t+i}$) and other influences. Subtracting the risk free rate $r^f_t$, $\Delta l^e_{t+i} = \Delta l_{t+i} - r^f_{t+i}$ is the excess rent growth rate, and $r^e_{t+i} = r_{t+i} - r^f_{t+i}$ is the excess return on housing. The operator $\tilde{E}_t$ symbolizes objective expectations of a variable based on information available at the end of
period $t$. Equation (6) decomposes the log PtR into three components: a time-varying deterministic term, discounted objective expectations of future rent growth rates and returns, and an error term.

The objective expectations $\bar{E}_t$ for all future excess rent growth rates and excess returns are calculated as forecasts from two alternative VAR regressions of order one. These comprise $y^{(1)} = (\eta_t, \Delta l_t^e, r_t^e)'$ and $y^{(2)} = (\eta_t, \Delta l_t^e, r_t^e, \pi_t)'$, where $\pi_t$ is the smoothed inflation. The smoothed inflation is used such that short term variations in the quarterly inflation are filtered out (e.g. Brunnermeier and Julliard; 2008). The composition of $y^{(1)}$ follows a long tradition proposed by Campbell and Shiller (1989), where three dimensional VAR models of dividend growth rates, stock returns, and price to dividend ratios are used to obtain the objective expectations of future dividend growth rates and stock returns. Including the smoothed inflation $\pi_t$ into the VAR model of $y^{(2)}$ is due to the concern that inflation can have effects on the expected future rent growth rates and returns on housing, as considered in Brunnermeier and Julliard (2008). Note that an unrestricted, VAR based determination of objective expectations applies irrespective of the potential of stochastic trends governing particular components in $y^{(1)}$ or $y^{(2)}$. The sample period in (6) starts in time $t_0$ which accounts for some burn-in period which is necessary to determine VAR predictions conditioning on historic data.

Providing the state equation, the financial state process $\rho_t$ is specified as a bounded random walk process (Cavaliere and Xu; 2011) with bounds at $[0, 1]$, i.e.

$$\rho_t = \rho_{t-1} + u_t, \quad \rho_{t_0} = \rho_0. \tag{7}$$

The disturbance term $u_t$ is decomposed as $u_t = e_t + \xi_t - \tilde{\xi}_t$, where $e_t \sim N(0, \sigma_e^2)$ and $\xi_t, \tilde{\xi}_t$ are non-negative processes such that $\xi_t > 0$ if and only if $\rho_{t-1} + e_t < 0$ and, similarly, $\tilde{\xi}_t > 0$ if and only if $\rho_{t-1} + e_t > 1$. At time $t = t_0$ $\rho_t$ is fixed to $\rho_0$, which is later treated as a parameter and subjected to estimation. To allow for a dynamic pattern of $\rho_t$, the state equation formalizes that this process exhibits a bounded stochastic trend with innovation variance $\sigma_u^2$. For given $\rho_t$ the in-sample determination of an implied model disturbance $\varepsilon_t$ is straightforward. It’s innovation variance is denoted by $\sigma_e^2$. Owing to the fact the that $\rho_t$ enters the observation equation in a highly non-linear manner, the model in (6) and (7) cannot be implemented by means of linear conditional modeling.
Consequently, the Kalman filter is not feasible to evaluate the model’s (log) density for given parameters. With known variance parameter $\sigma^2$, however, the evaluation of the model’s log density is straightforward for a given time path of $\rho_t, t = t_0, \ldots, T$. Since this process is not observable but straightforwardly specified in (7), the so-called particle filter allows a Monte Carlo based evaluation of the log-likelihood function for given parameters in $\theta = (\rho_0, \sigma^2_u, \sigma^2_{\epsilon})$.

Owing to persistence patterns characterizing actual quotes of the log PtR one might doubt the iid assumption for the equilibrium errors in (6) and prefer a dynamic, moving average say, pattern of $\epsilon_t$. In fact, empirical estimates $\hat{\epsilon}_t$ strongly hint at serial correlation of the underlying process. In this case the estimated model parameter $\sigma^2_\epsilon$ could be seen as an unconditional variance. Regarding this unconditional variance it turns out that it is only mildly affected by the selection of the remaining parameters in $\theta$ which are of the core interest in our empirical setup. As a particular rival model we also consider a degenerated state-space model with constant $\rho$, for which $\sigma^2_u = 0$ is imposed. It will be of particular interest to evaluate the approximation losses in terms of the Gaussian log-likelihood when switching from the dynamic state-space model to its degenerate counterpart.

We employ the particle filter (Del Moral; 1996) as described with resampling in Cappé, Godsill and Moulines (2007) for likelihood evaluation. Model parameters in $\theta = (\rho_0, \sigma^2_u, \sigma^2_{\epsilon})$ are determined by means of a grid search. For those parameter combinations obtaining the maximum of the Gaussian log-likelihood, $\theta_{opt}$, implied time paths $\hat{\rho}_t, t = t_0, \ldots, T$, are determined by averaging over simulated particles. Noting the low dimension of $\theta$, the number of particles is relatively small, $N = 2000$, however, we perform the grid search multiple (i.e. 10) times to check if results are robust or suffer from prohibitive Monte Carlo errors. The adopted implementation outlined in Cappé, Godsill and Moulines (2007) (Algorithm 3, with using $\rho_t \sim N(\rho_{t-1}, \sigma^2_u)$ as importance distribution) is adapted for our purposes. Accordingly, the following two step algorithm is implemented:

**Algorithm 1** Step (1): Initialization ($t = 1$). Sample $N$ particles $\hat{\rho}^{(i)}_1 \sim$
$N(\rho_0, \sigma_x^2)$, $i = 1, \ldots, N$, and determine importance weights

$$\hat{w}_1^{(i)} = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp \left( -\frac{1}{2} \left( \frac{\varepsilon^{(i)}}{\sigma_x} \right)^2 \right).$$

Normalized weights are obtained as

$$w_1^{(i)} = \frac{\hat{w}_1^{(i)}}{\sum_i \hat{w}_1^{(i)}}.$$

Step (2): Iteration ($t = 2, \ldots, T$).

a: Select $N$ particles according to weights $w_{t-1}^{(i)}$. Set accordingly $\hat{\rho}_{t-1}^{(i)} = \hat{\rho}_{t-1}^{(i)}$ and $w_{t-1}^{(i)} = 1/N$ (resampling).

b: For all particles draw

$$\hat{\rho}_t^{(i)} \sim N(\rho_{t-1}^{(i)}, \sigma_x^2), \ i = 1, \ldots, N,$$

and determine raw weights

$$\hat{w}_t^{(i)} = w_{t-1}^{(i)} \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp \left( -\frac{1}{2} \left( \frac{\varepsilon_t^{(i)}}{\sigma_x} \right)^2 \right).$$

c: Normalize weights

$$w_t^{(i)} = \frac{\hat{w}_t^{(i)}}{\sum_i \hat{w}_t^{(i)}}.$$

d: go back to step ‘a’.

Averaging over weighted draws obtains estimates of the contribution of $\varepsilon_t$ to the Gaussian log-likelihood and, more interestingly, time dependent estimates of $\rho_t$, i.e., $\hat{\rho}_t = \frac{1}{N} \sum_{i=1}^N \hat{\rho}_t^{(i)}, \ t = 1, \ldots, T.$
4 Model evaluation

In this section, the proposed state space model is evaluated with quarterly US data from 1975:Q1 to 2009:Q2. The FHFA housing price index and the rent of primary residence as a component of the CPI are used to obtain $\eta_t$, $\Delta l_t$, and $r_t$. The 10-year Treasury Bill rate is adopted for $r_{f}^t$ and smoothed inflation $\pi_t$ is calculated from the CPI excluding shelter. Note that a long-term instead of short-term risk free rate is considered to reflect the long-run holding time period of a home in general. Detailed descriptions of the data are provided in the Appendix. The first 30 observations are used to initiate the recursive VAR modeling and the provision of multistep forecasts. Three conclusions can be drawn from our analysis.

Firstly, the VAR model including inflation ($y^{(2)}$) has a better performance than the one without inflation ($y^{(1)}$). As can be seen from Table 2 the log-likelihood of the former (478.45) is about 35% higher than the log-likelihood of the latter (354.33). This evidence supports the view that inflation influences the agents’ expectation of rent growth rates and returns. For the further analysis in the next section, we mostly consider the estimates based on the VAR model with inflation ($y^{(2)}$).

Secondly, the estimated time path of $\rho_t$ is clearly time varying. Figure 3 illustrates the estimated time path of $\rho_t$ for the two alternative VAR models. Both paths of $\hat{\rho}_t$ are time varying and different from the constant $\rho$ (red dashed line), which is the observed sample mean of $P_t/(P_t + D_t)$. Confirming the visual impression, it can be observed from Table 2 that, according to log-likelihood statistics, the model with the time varying $\rho_t$ is always strongly preferred over its constant parameter counterpart. When the VAR for $y^{(2)}$ is considered, the log-likelihood value of the time varying $\rho_t$ model (478.45) is about 15% higher than the one from the constant $\rho$ model (416.26). Although one might question the validity of common likelihood (ratio) comparisons of rival models in the present context, it is most unlikely that the reported log-likelihood improvement accords with repeated experiments under the null hypothesis of a constant $\rho$ model.

The increase in the financial state $\rho_t$ has a strong and complex impact on the log PtR. Equation (6) shows that not only the deterministic term but also the sum of future discounted cash flows increases, ceteris paribus.
The degree of increases in the log PtR depends on the expectation of future rent growth rates and returns at a given time point. Consider 1982:Q3 for a simplified example. At this time point the estimated state variable $\hat{\rho}_t$ is 0.984 and the observed risk adjusted rent growth rate $\Delta l_t - r_t$ is around 0.006. Assuming a constant future risk adjusted rent growth rate of 0.006, if the state variable increased to 0.986, the resulting log PtR would increase about 3%. This accounts for about 40% of the observed increase in the log PtR from 1983:Q2 to 2006:Q4.

Moreover, the estimated mispricing errors from a constant $\rho$ model may lead to biased conclusions. Figure 4 displays $\hat{\varepsilon}_t$ (blue solid line) from the state space model with time varying $\rho_t$ along with the corresponding process (blue dotted line) obtained when the sample average $\bar{\rho}$, the mean of $P_t/(P_t + D_t)$, is used to substitute $\rho_t$ in equation (6). While the mispricing errors with the time varying $\rho_t$ are mostly rather small, their counterparts from the constant $\rho$ model have varied between -0.04 to 0 in the 1980s and built up to 0.04 during the recent period of excess pricing in the housing market. From 2004:4 to 2006:3, implied residuals from the constant $\rho$ model have increased by about 50 times. Over the same time period, the estimated errors of the time varying $\rho_t$ model have only doubled and reached up to 0.0042. The constant $\rho$ model cannot distinguish the movements of mispricing from changes of the long-term PtR, and, thus, obtains overly large mispricing errors.

To summarize, the estimated financial state ($\hat{\rho}_t$) indicating the long-term state of the log PtR is clearly time-varying. When this time variation is taken into account, the estimated error terms are at a markedly smaller scale in comparison with residual terms derived from the constant $\rho$ model. In the next section, we investigate the potential influence of credit market conditions on the long-run log PtR through the financial state $\rho_t$.

5 Cointegration analysis

In this section we investigate the cointegration relation among the estimated financial state $\hat{\rho}_t$ and it’s potential determinants in credit markets. The financial state $\hat{\rho}_t$ is determined by means of particle filtering within an asset pricing system that has not included measures characterizing
credit market conditions. VECMs are applied due to the non-stationarity of the variables, their joint endogeneity and the potential of common stochastic trends. Figure 5 illustrates the considered time series.

5.1 Preliminary analysis

We discuss first the data and then employ unit root tests and cointegrating rank tests for the financial state \( \hat{\pi}_t \), the real mortgage rate \( \text{rm}_t \), and the securitization ratio \( s_t \).

To obtain the real mortgage rate, we use nominal contract rates on the 30-year fixed-rate conventional home mortgage adjusted for inflation expectations. In the related literature, a long-term treasury bond rate rather than the mortgage rate has often been used to study the influence of interest rates on the housing market. The reason for this choice is to isolate endogenous fluctuations in market interest rates due to the housing market since OLS estimation cannot cope adequately with the endogeneity (e.g. Glaeser, Gottlieb and Gyourko; 2010). Since the VECM explicitly considers the effects of endogenous variables on each other, we use the mortgage rate to incorporate the potential dynamics in the data. With regard to inflation expectation, we draw the data from the Survey of Professional Forecasters at the Federal Reserve Bank of Philadelphia\(^1\). It is the mean of forecasts for the annual average rate of CPI inflation over the next 10 years. Data are available over the period from 1991:4 to 2009:2. As can be seen from Figure 5, inflation expectations have been very stable and fluctuated around 2.5% since 1998.

Reliable aggregate data that measure directly the underwriting standards are not publicly available. Both Federal banking regulators and the Office of the Comptroller of the Currency conduct surveys to ask about banks’ underwriting standards. However, these surveys don’t include the non-bank financial sectors which have been largely involved in underwriting subprime mortgages. Noting from Keys et al. (2010) that increasing securitization practices and decreasing underwriting standards are highly related to each other we approximate the latter by the former. More precisely, we measure the securitization practices by means of the share of

\(^1\)This Survey instead of the Livingston and Michigan Survey of inflation expectations is chosen since it provides inflation expectations at the quarterly frequency over a long horizon.
the outstanding home mortgages held by private issuers of asset backed securities, called the securitization ratio henceforth. Data are collected from the flow of funds accounts released by the Board of Governors of the Fed.

Table 3 provides the unit root test statistics for the three considered variables: the financial state \( \hat{\rho}_t \), the real mortgage rate \( rm_t \), and the securitization ratio \( s_t \). The time periods used are the longest available periods for each variable. Results from alternative unit root tests are consistent with each other except for a few cases. There is strong evidence supporting the view that all investigated processes are diagnosed to be integrated of order one. Given the non-stationarity of these time series, we continue with tests for the cointegrating rank.

The cointegration rank among \( \hat{\rho}_t \), \( rm_t \), and \( s_t \) is tested for the common sample period from 1991:Q4 to 2009:Q2. Table 4 reports the results from Johansen trace tests. Since AIC suggests 3 as the lag order for the differences and SC is minimized for lag order 1, lagged differences from 1 to 3 are considered. The overall evidence suggests that there is at least one cointegration relation.

As the next step, we adopt the so called S2S approach to estimate the VECMs (Ahn and Reinsel; 1990). Brüggemann and Lütkepohl (2005) show that this estimator does not produce the outliers as sometimes seen when following ML estimation, particularly when conditioning on small samples. Furthermore, to reduce the number of parameters and the estimation uncertainty, we apply a subset procedure. The cointegrating vector is estimated first. Then linear restrictions on the parameters that characterize short term dynamics are imposed. Explanatory variables with smallest absolute \( t \)-ratios are sequentially deleted until all \( t \)-ratios exceed 1.96 in absolute value. At each step, the entire system is estimated again and new \( t \)-ratios are updated within the reduced model. Estimation results are documented in the next subsection.

5.2 Results

First, the signs of the estimated cointegration coefficients are consistent with the theory. While real mortgages rates have a significantly negative effect on the long-term state of the log PtR, the securitization ratio has a significantly positive effect, as can be seen from the first row of Table 5.
for the entire available sample period from 1991:Q4 to 2009:Q2. Lower real mortgage rates reduce financial costs of mortgage loans and thereby stimulate the demand for houses. Higher proportions of the home mortgage funds from securitization activities may stimulate the credit supply as a result of agency problems along the securitization chain. Through the new financing model of mortgage funds, the cheap credit has led to the increases in the long-term PtR.

The results of the cointegration relation are robust. Consider, for instance, an alternative sample period of 1996:Q1 to 2006:Q4, which is characterized by most intensive accelerations of house prices in relation to rents. As can be seen from the second row of Table 5, subsample results allow the same qualitative conclusions. In addition, results from the portmanteau tests confirm that there is no significant autocorrelation in the residuals.

Moreover, the estimated cointegrating relation bears the interpretation of a housing market equation. Both real mortgage rates and the securitization are weakly exogenous towards their cointegration relation with the financial state in housing markets. The estimated adjustment coefficients for real mortgage rates and securitization ratios are not significantly different from zero. Credit market conditions such as real mortgage rates and securitization cause the variation in the long-term state of the housing markets, but not opposite. To provide a robust analysis of the adjustment dynamics of the variables, we consider recursive estimates of adjustment coefficients. Given the estimated cointegration coefficients from the full sample, recursive estimates of the remaining model parameters are obtained from recursively samples beginning in 1996:Q4 and ending from 1996:Q4 to 2009:Q2. Figure 6 displays these estimates jointly with respective 95% confidence intervals.

As can be seen from Figure 6, the only adjustment coefficient, which differs significantly from zero, is obtained for the state variable $\hat{p}_t$. In particular, its adjustment towards the cointegration relation becomes significant since 2002, and reaches a level of about -0.12 at the end of the sample. It takes the long-run log PtR about 2 years to fully adjust to its equilibrium level with the real mortgage rates and securitization ratios. In contrast, real mortgage rates are not influenced by deviations from the cointegrating relation. Its adjustment coefficient is never significant over the entire recursion. The securitization ratio is also weakly exogenous to
the cointegration relation. Nevertheless, around 1999, and between 2002 and 2004, there is mild evidence for some significant adjustment towards the increasing long-run PtR.

Furthermore, we conduct a forecast error impulse response analysis to have a more comprehensive picture of the impact of a shock in credit markets on the state variable in housing markets. The relationships among the variables in VECMs are more complex as indicated by the cointegration parameters due to jointly endogenous dynamics. In the impulse response analysis, the expected response of the state variable is traced out over the next 5 years given a one time innovation of size one standard deviation in the state variable, the real mortgage rates, and the securitization ratio. Figure 7 illustrates these impulse responses along with 95% bootstrap confidence intervals.

It is striking to observe that the securitization ratio has the highest impact on long-term state of the log PtR. After five years, the impact of a standardized shock in the securitization ratio on the state variable is 3 times larger than the impact of a standardized shock in real mortgage rates, and 63 times larger than a standardized shock in the state variable itself. This evidence supports the view that the securitization of the residential mortgage loans has played the most important role in the recent increases of the house price relative to rent. In addition, changes in the expectations of home buyers about the future return and rent growth rates, which might also affect the long-term log PtR, have only temporary effects over time. As can be seen from Figure 7, the effect of a shock in the state variable itself decreases slowly over time, while shocks in the real mortgage rates and the securitization have persistent effects on the state variable. Besides, the impulse responses from the real mortgage rates are obtained with low precisions. The corresponding 95% bootstrap confidence intervals are very wide and close to the zero line.

The results from the forecast error impulse response analysis are obtained with only one shock in one variable at a time. If the shocks are instantaneously correlated, this analysis might only provide a partial picture. Table 6 provides results from tests for instantaneous causality. Shocks in the securitization ratio do not instantaneously cause shocks in the state variable or the real mortgage rate. The correlations between estimated residuals from the securitization ratio and those from the state variable and real mortgage rates are 0.087 and 0.015 accordingly. Simi-
larly, shocks in the real mortgage rate are not instantaneously related to the shocks in the state variable and the securitization at the 5% significance level. Nevertheless, shocks in the state variable do instantaneously cause shocks in the remaining two variables. This result is due to the fact that the correlation between estimated residuals from the state variable and those from real mortgage rates is about $-0.289$. Therefore, it is likely that shocks in real mortgage rates and the state variable happen simultaneously. However, even when the instantaneous correlation between shocks in real mortgage rates and the state variable is taken into account by means of a structural VECM, the resulting forecast error impulse responses are similar to those in Figure 7. The reason is that not only the adjustment coefficient but also all short run coefficients in the equation of the real mortgage rates are not significantly different from zero.

6 Conclusions

This paper focuses on the linkage between the housing markets and credit markets in the US from the early 1990s to 2009. The effect of the real mortgage rates and securitization activities on house prices relative to the rent is investigated.

We propose a state space model with a time-varying state variable reflecting the long-term price to rent ratio. Changes in the long-term state of the price to rent ratio may occur if there are changes in long-run rent growth rates and returns on houses. An increasing long-term price to rent ratio from the early 1990s to 2007 is supported by the data. In particular, we show that the present value model neglecting the time variation in the long-term state leads to a lower log-likelihood valuation and overly large mispricing terms in the model.

Recent increases in the long-term house price to rent ratio have been caused by decreasing real mortgage rates and increasing securitization activities, especially since 2002. Lower real mortgage rates decrease the financial costs of mortgage loans, and increase the demand for houses. Increases in securitization activities have played the most important role to explain the upward trend in the long-term price to rent ratio. The effect of a standardized shock in securitization activities is 3 times larger than
the effect of a standardized shock in the real mortgage rates. Through
securitization, the new financing model of mortgage funds has resulted
in cheap credit for the home buyer, and thus, increased prices.

It is also worth mentioning that we have considered to investigate
the long-term house price to rent ratio in different metropolitan areas
in the US. Nevertheless, such a study has been incapacitated by the
unavailability of long-span quarterly data of rents at the local level.

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**Appendix**

**Data description**

Quarterly US data from period of 1975:Q1 to 2009:Q2 for the housing price index, the rent index, T-bill rates and the inflation are considered. We use the FHFA (formerly OFHEO) housing price index, which provides the longest available quarterly time series of housing prices. The rent index is the rent of primary residence as a component of the consumer price index released by the U.S. Bureau of Labor Statistics (BLS). To obtain the log PtR, the housing price index is scaled so that the mean of the log PtR is about 4.1581, as reported by Ayuso and Restoy (2006) for the sample period of 1987 to 2003. As the long-term risk free rate, we
use time series of the 10-Year Treasury Bill rate provided by the Board of Governors of the Federal Reserve System. The consumer price index (CPI) excluding shelter from BLS is used to obtain time series of the smoothed inflation. Specifically, exponentially weighted moving averages of quarterly inflation are determined with a smoothing time period of 16 quarters.

To analyse the movement of the PtR, real mortgage rates and securitization are employed. The considered nominal mortgage rate is the contract rate on 30-year fixed-rate conventional home mortgage. Data is provided by the Board of Governors of the Fed. The real mortgage rate is obtained by deflating the nominal mortgage rate with inflation expectations as published in the Survey of Professional Forecasters at the Federal Reserve Bank of Philadelphia. It is the mean of forecasts for the annual average rate of CPI inflation over the next 10 years. Data are available for the period 1991:4 to 2009:2. To measure the securitization activities, we use the share of the home mortgage held by the private issuers of asset backed securities. The related data are from the flow of funds accounts released by the Board of Governors of the Federal Reserve System. Notably, the data of total mortgage held by the issuers of asset backed securities is only available since 1984.
### Table 1: Unit root tests for the US log PtR

<table>
<thead>
<tr>
<th></th>
<th>$ADF_t$</th>
<th>$PP_t$</th>
<th>$DF_{GLS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test statistics</td>
<td>-2.36</td>
<td>-2.15</td>
<td>-1.45</td>
</tr>
<tr>
<td>Critical values at 10%</td>
<td>-2.58</td>
<td>-2.58</td>
<td>-1.62</td>
</tr>
</tbody>
</table>

Notes: A constant is included, and SC lag length selection criterion is employed to obtain the above test statistics. $ADF_t$ refers to the Augmented Dickey-Fuller $t$ test. For $PP_t$, the $t$ test statistic considered in Phillips and Perron (1988), the spectral AR estimator is used to calculate the long run variance. $DF_{GLS}$ refers to the modified Dickey-Fuller $t$ test proposed by Elliott, Rothenberg and Stock (1996). The time period is from 1975:2 to 2009:2, the total available period.

### Table 2: Parameter estimates and model evaluations

<table>
<thead>
<tr>
<th>VAR</th>
<th>Time varying $\rho$</th>
<th>Constant $\rho$ ($\sigma_e = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_0$</td>
<td>$\sigma_x$</td>
</tr>
<tr>
<td>$y^{(1)}$</td>
<td>0.984</td>
<td>8.27E-03</td>
</tr>
<tr>
<td>$y^{(2)}$</td>
<td>0.984</td>
<td>2.61E-03</td>
</tr>
</tbody>
</table>

Notes: This table documents core parameter estimates ($\rho_0$ and standard deviations) and model diagnostics for the two dynamic specifications and their time invariant counterpart. The time period is from 1982:3 to 2009:2. The first 30 observations from 1975:2 to 1982:2 are used to initiate recursive VAR forecasting to determine objective expectations $\tilde{E}_t$ in (6).
Table 3: Unit root test statistics

<table>
<thead>
<tr>
<th></th>
<th>( ADF_t )</th>
<th>( PP_t )</th>
<th>( DF_{GLS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\rho}_t )</td>
<td>1.29</td>
<td>1.56</td>
<td>1.09</td>
</tr>
<tr>
<td>( \Delta \hat{\rho}_t )</td>
<td>-5.22***</td>
<td>-5.22***</td>
<td>-5.16***</td>
</tr>
<tr>
<td>( \hat{rm}_t )</td>
<td>-2.05</td>
<td>-2.05</td>
<td>-1.92*</td>
</tr>
<tr>
<td>( \Delta \hat{rm}_t )</td>
<td>-6.93***</td>
<td>-8.41***</td>
<td>-2.30**</td>
</tr>
<tr>
<td>( \hat{s}_t )</td>
<td>-2.05</td>
<td>-2.39</td>
<td>-1.47</td>
</tr>
<tr>
<td>( \Delta \hat{s}_t )</td>
<td>-2.85*</td>
<td>-2.69*</td>
<td>-2.80***</td>
</tr>
</tbody>
</table>

Notes: Test statistics being significant at 10%, 5% and 1% are indicated with *, **, and ***, respectively. The financial state is denoted by \( \hat{\rho}_t \). \( \hat{rm}_t \) is the real mortgage rate. The securitization ratio is represented by \( \hat{s}_t \). To provide an overview, we use longest available periods for each variable. The sample period for \( \hat{\rho}_t, \hat{rm}_t \) and \( \hat{s}_t \) are 1982:Q3 to 2009:Q2, 1991:Q4 to 2009:Q2, and 1984:Q4 to 2009:Q2, respectively. See previous notes in Table 1 for detailed descriptions of the unit root tests.

Table 4: Johansen trace tests for \((\hat{\rho}_t, \hat{rm}_t, \hat{s}_t)\)

<table>
<thead>
<tr>
<th>Lagged differences</th>
<th>( H_0 )</th>
<th>Test statistic</th>
<th>( p)-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( r = 0 )</td>
<td>41.12</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>( r = 1 )</td>
<td>18.11</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>( r = 2 )</td>
<td>2.37</td>
<td>0.70</td>
</tr>
<tr>
<td>2</td>
<td>( r = 0 )</td>
<td>39.98</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>( r = 1 )</td>
<td>13.39</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>( r = 2 )</td>
<td>2.58</td>
<td>0.67</td>
</tr>
<tr>
<td>3</td>
<td>( r = 0 )</td>
<td>67.70</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>( r = 1 )</td>
<td>21.87</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>( r = 2 )</td>
<td>2.19</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Notes: Testing the cointegration rank for the financial state \((\hat{\rho}_t)\), the real mortgage rate \((\hat{rm}_t)\), and the securitization ratio \((\hat{s}_t)\). A constant is included. The sample period is from 1991:Q4 to 2009:Q2, the available common sample period for all variables.
Table 5: Cointegration parameters: \( \hat{\rho}_t = \beta_1 \hat{r}m_t + \beta_2 s_t \)

<table>
<thead>
<tr>
<th>Time period</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991:Q4 - 2009:Q2</td>
<td>-0.029</td>
<td>0.018</td>
</tr>
<tr>
<td></td>
<td>(-4.396)</td>
<td>(17.009)</td>
</tr>
<tr>
<td>1996:Q1 - 2006:Q4</td>
<td>-0.0051</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>(-2.124)</td>
<td>(12.178)</td>
</tr>
</tbody>
</table>

\( p \)-values for Portmanteau tests

<table>
<thead>
<tr>
<th>Time period</th>
<th>lag order 4</th>
<th>lag order 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991:Q4 - 2009:Q2</td>
<td>0.184</td>
<td>0.147</td>
</tr>
<tr>
<td>1996:Q1 - 2006:Q4</td>
<td>0.798</td>
<td>0.996</td>
</tr>
</tbody>
</table>

Notes: A constant is included in the estimation. S2S approach is used to estimate the cointegration relation among the financial state (\( \hat{\rho}_t \)), the real mortgage rate (\( \hat{r}m_t \)), and the securitization ratio (\( s_t \)) (\( t \)-statistics in parentheses). For the time period of 1991:Q4 to 2009:Q2, the lag length of 3 is chosen. For period of 1996:Q1 to 2006:Q4, one lag is considered. The lag length is chosen under consideration of diagnostics of residual autocorrelation.

Table 6: Wald tests for instantaneous causality

\( H_0 \): no instantaneous causality between \( \hat{\rho}_t \) and \( (\hat{r}m_t, s_t)' \)

<table>
<thead>
<tr>
<th>Test statistic</th>
<th>( \hat{\rho}_t ) and ( (\hat{r}m_t, s_t)' )</th>
<th>( \hat{r}m_t ) and ( (\hat{\rho}_t, s_t)' )</th>
<th>( s_t ) and ( (\hat{\rho}_t, \hat{r}m_t)' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )-value</td>
<td>7.30</td>
<td>5.53</td>
<td>4.12</td>
</tr>
<tr>
<td></td>
<td>0.03</td>
<td>0.06</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Notes: The test statistic is \( \chi^2 \) distributed with 2 degrees of freedom. The financial state is denoted by \( \hat{\rho}_t \). \( \hat{r}m_t \) is the real mortgage rate. The securitization ratio is represented by \( s_t \).
Figure 1: The log housing price to rent ratio from 1975:Q2 to 2009:Q2 for the US, the total available sample period.
Figure 2: The approximation error for the period of 1991:Q4 to 2004:Q2 from the present value model with the fixed steady state in (3). The starting period 1991:Q4 is the same as the one for the final analysis of the effect of credit market conditions on housing markets. The ending period 2004:Q2 is chosen so that there are at least 20 observations available for the smallest forward looking time period.
Figure 3: Estimated $\rho_t$ for the available time period. The first 30 observations from 1975:Q2 to 1982:Q2 are used to initiate recursive VAR forecasting for the estimation.

Figure 4: Estimated errors $\varepsilon_t$ with the log PtR.
Figure 5: The considered variables for the available common time period.
Figure 7: Impulse responses of the financial state variable $\rho_t$ with respect to an innovations of size one standard deviation in the state variable, real mortgage rates, and the securitization ratio. The dashed lines are the 95% Efron (bootstrap) confidence intervals based on 299 bootstrap replications.