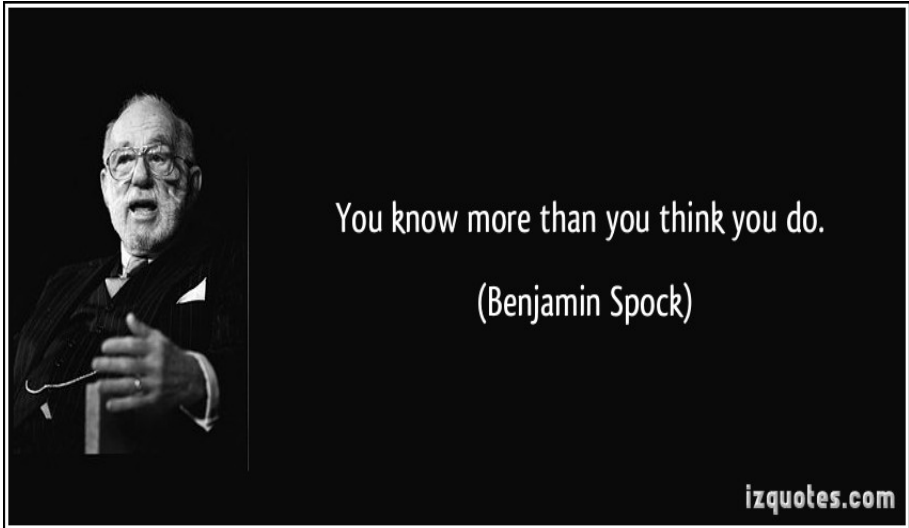


Stochastic Processes

MATH5835, P. Del Moral

UNSW, School of Mathematics & Statistics

Lectures Notes No 12



– *Dr. Benjamin Spock (1903-1998)*

MARKOV CHAINS (discrete time)



*** among the first 4 points

- ▶ Topological aspects

Recurrent/transient random walks ($d = 1, 2, 3$)

MARKOV CHAINS (discrete time)



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Matrices, integral operators, evolution semigroups.

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- ▶ Stability properties:
Minorization condition, invariant measures

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Coupling techniques, stopping/coupling times.



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- ▶ Stability properties:
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- ▶ Stochastic tools:
Coupling techniques, stopping/coupling times.
- ▶ Analytic tools:
Spectral techniques, total variation, Dobrushin contraction, Perron Frobenius, Wasserstein distance.

... and MARTINGALES



*** among the first 3 points

- ▶ Decompositions (of any stoch proc.)
- ▶ Predictable angle brackets
- ▶ Hitting times: gambling, ruin processes.
- ▶ Transformations: stopping, driving,...
- ▶ Poisson equation, ergodic theorem.

NONLINEAR MARKOV MODELS



- ▶ Self-interacting models:

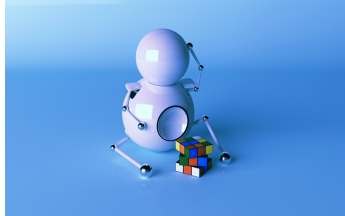
Reinforcement processes, a toy model (\simeq slow \ln)

- ▶ Mean field particle models:

Particle simulation, genetic dynamic models

*** among the first 5 points

- ▶ Google page rank.
- ▶ Coupling/Meeting times.
- ▶ Spatial branching, migration, birth and death.
- ▶ Metropolis Hasting model.
- ▶ Gibbs-Glauber model.
- ▶ Cellular dynamic models.
- ▶ Molecular dynamics, rare event simulation.
- ▶ Piecewise deterministic models.



- ▶ Metropolis Hasting model.
- ▶ Gibbs-Glauber model.
- ▶ Reinforcement processes (toy model).
- ▶ Robbins Monro model.
- ▶ Stochastic gradient.
- ▶ Iterated random functions.
- ▶ Kalman/particle filters

MARKOV PROCESSES (cont. time)



★★ (with no jumps)

- ▶ Typical models:

Brownian motion, spatial-jump processes, jump-diffusions.

- ▶ Stochastic modeling:

infinitesimal generators, evolution semigroups

- ▶ Stochastic calculus

Ito-Doebelin-Taylor formula, martingale decompositions, angle brackets

ILLUSTRATIONS (continuous time)



★★ among the first 4 points

- ▶ Heat equation.
- ▶ Geometric Brownian motion.
- ▶ Ornstein-Uhlenbeck model.
- ▶ Fokker-Planck equation.
- ▶ Fluid models.
- ▶ Mathematical finance (backward martingales, option pricing, Black-Scholes)



**KEEP
CALM
AND
EXAM
SUCCESS**