Particle methods in statistical learning and risk analysis

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UNSW, School of Mathematics and Statistics

2014 BIG DATA SCIENCE Workshop, UNSW july 1st 2014

Some hyper-refs

- Feynman-Kac formulae, Genealogical & Interacting Particle Systems with appl., Springer (2004)
- Mean field simulation for Monte Carlo integration. Chapman - Hall (2013) [+ Refs]
- More references on website http://web.maths.unsw.edu.au/∼peterdel-moral/simulinks.html [+ Links]

Stochastic particle sampling methods

Sampling conditional distributions

Some concrete applications

Introduction

Stochastic particle methods
=
Universal adaptive sampling technique

2 types of stochastic interacting particle models:

- Diffusive particle models with mean field drifts
  [McKean-Vlasov style]
- Branching and Interacting jump particle models
  [Boltzmann & Feynman-Kac style]

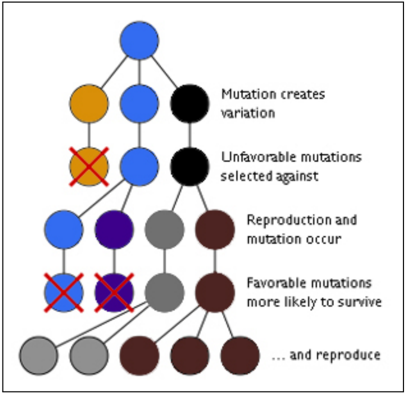
Workshop $\subset$ Interacting jumps models

- **Interacting jumps = Recycling transitions =** 

- **Discrete time models ($\Leftrightarrow$ geometric rejection/jump times)**

Genetic type interacting particle models

- Mutation-Proposals w.r.t. Markov transitions $X_{n-1} \rightsquigarrow X_n \in E_n$.
- Selection-Rejection-Recycling w.r.t. potential/fitness function G_n.



Equivalent particle algorithms

Sequential Monte Carlo	Sampling	Resampling
Particle Filters	Prediction	Updating
Genetic Algorithms	Mutation	Selection
Evolutionary Population	Exploration	Branching-selection
Diffusion Monte Carlo	Free evolutions	Absorption
Quantum Monte Carlo	Walkers motions	Reconfiguration
Sampling Algorithms	Transition proposals	Accept-reject-recycle

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Convergence/Performance analysis : CLT, LDP, $\mathbb{L}_p$-estimates, Empirical processes, Moderate deviations, propagations of chaos, unif cv w.r.t. time, new stochastic models....

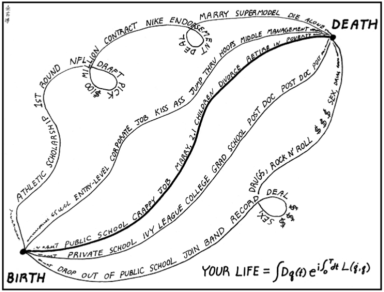
Concentration analysis = Exponential deviation proba. estimates

A single stochastic model

Particle interpretation of Feynman-Kac path integrals

=

ANY CONDITIONAL DISTRIBUTION



The Path Integral Formulation of Your Life

Some conditional distributions

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$$\textbf{Law}\left(\textbf{stoch. process } X \mid \textbf{partial observations } Y\right)$$

Ex.: radar/sonar/data assimilation/multiple object tracking

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- **Black box - risk analysis**

  $$\textbf{Law}\,(\textbf{Inputs/process} \mid \textbf{critical ouput/rare event})$$

  Ex.: Uncertainty propagations, sensitivity analysis, ruin processes, rare event simulation

- **Global optimization**: interacting simulated annealing, ...

Particle algorithms

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$\simeq_N = \textit{number of samples/computational power} \uparrow \infty$

Empirical measure: $\frac{1}{N} \sum_{1 \leq i \leq N} \delta_{X^i}$

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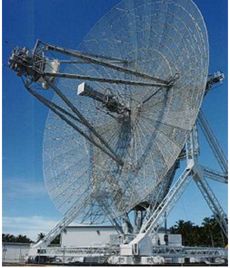
Signal processing & filtering models



Law (**Markov process** X | **Noisy & Partial observations** Y)

- **Signal** X **:** target evolution (missile, plane, robot, vehicle, image contours), forecasting models, assets volatility, speech signals, ...
- **Observation** Y **:** Radar/Sonar/Gps sensors, financial assets prices, image processing, audio receivers, statistical data measurements, ...

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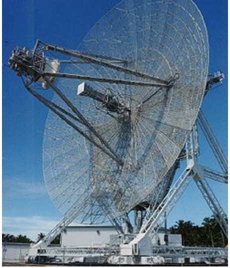

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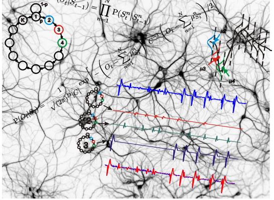
Industrial transferts & some research funding

- PhD/LAAS-CNRS-French Defense : 1989-1995.
- ANR Propagation 2009-2012: 2.3 M euros.
- DGA, DCNS-SIS, THALES, ASTRIUM (Radar/Sonar/GPS).
- Météo France (2000-2010) : Forecasting and turbulent fluid models

Hidden Markov chains problems



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$$\textbf{Law} \left(\text{fixed parameter } \Theta \;\middle|\; \text{Noisy \& Partial observations } Y^{\Theta} \right)$$

- **Parameter Θ :** kinetic model unknown parameters, statistical parameters (signal/sensors), hypothesis testing, ..
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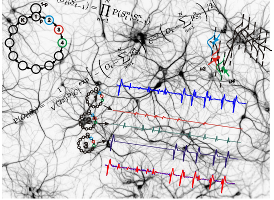

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Industrial transferts & some research funding

- Biips Software development (2006-2014).
- Epidemiology : ANR Viroscopy 2009-2012 (40KE).
- Food risk, Eco-Microbiology : ARC EPS (2011) (80KE+Post-Doc).

Uncertainty propagations in numerical codes

Law (**Inputs** $\mathcal{I}$ | **Outputs** $\mathcal{O} = C(\mathcal{I})$ $\in$ **Reference or Critical event**)

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Industrial transferts & some research funding

- ARC INRIA RARE (2009 - 7 INRIA teams collaboration project).
- Satellite debris tracking/control : ONERA & CNES (40KE+PhD).
- Nuclear plant security : EDF (40KE+PhD)
- Offshore structures reliability : IFREMER (20KE).