



BiiPS software: statistical learning in finance

ALEA project-team, Inria Bordeaux – Sud-Ouest

Context

The last two decades have seen a rapid development of increasingly **realistic and sophisticated stochastic models and methods** for **pricing, hedging and risk management in rapidly growing markets.**

Problem

Modern finance is becoming **increasingly technical**,
requiring the use of **advanced stochastic algorithms**.

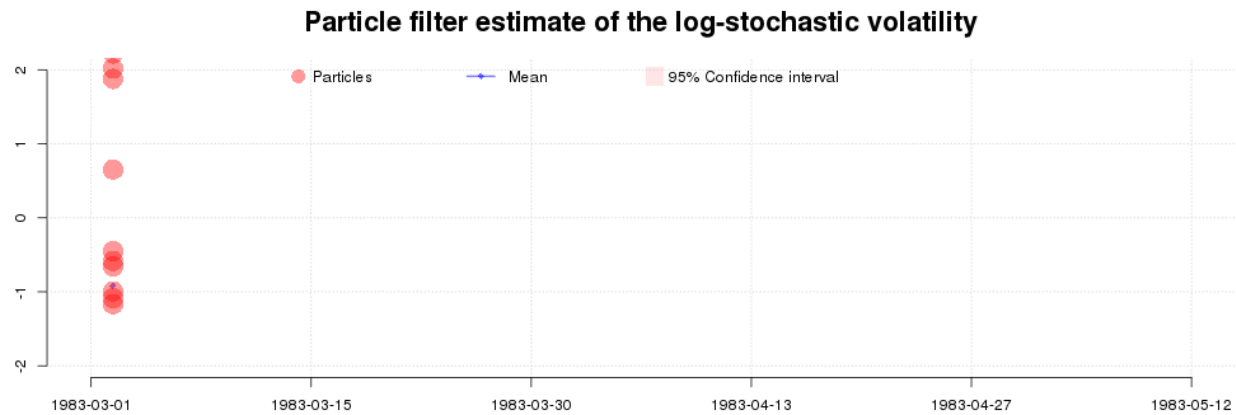
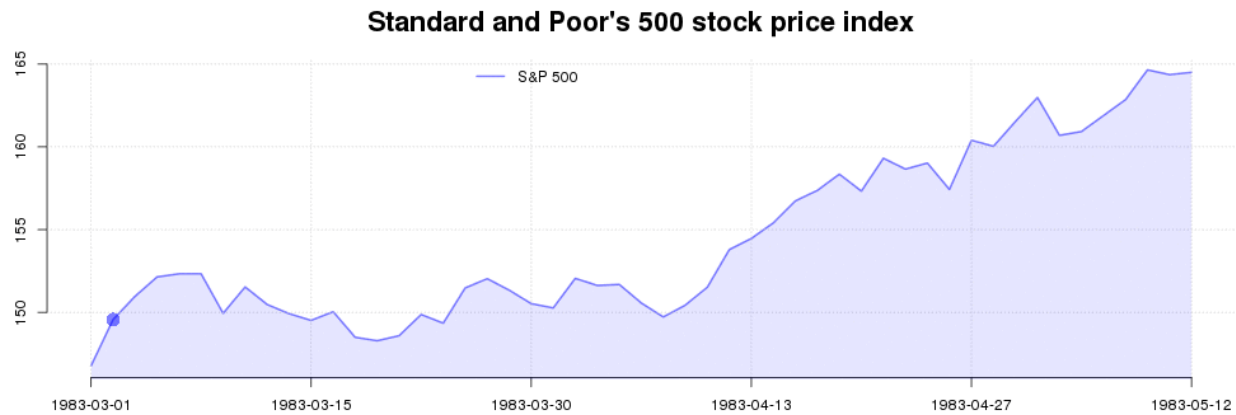
1

Advanced Particle and Sequential Monte Carlo Algorithms

Our expertise

As a pioneer in the field of **new adaptive and interacting particle algorithm** and **SMC methods**, the ALEA project-team is working on a **great variety of numerical applications**.

Particle algorithms illustrated



Numerical applications

- Parameter estimation
- Calibration of valuation models
- Derivative pricing
- Sensitivity analysis
- Hedging in incomplete markets
- Credit risk
- Risk and uncertainty quantification
- Portfolio optimization
- Etc.

Team members implied in financial mathematics:

P. Del Moral, P. Hu, A. Richou, F. Caron, A. Todeschini, C. Li (Master X2009).

2

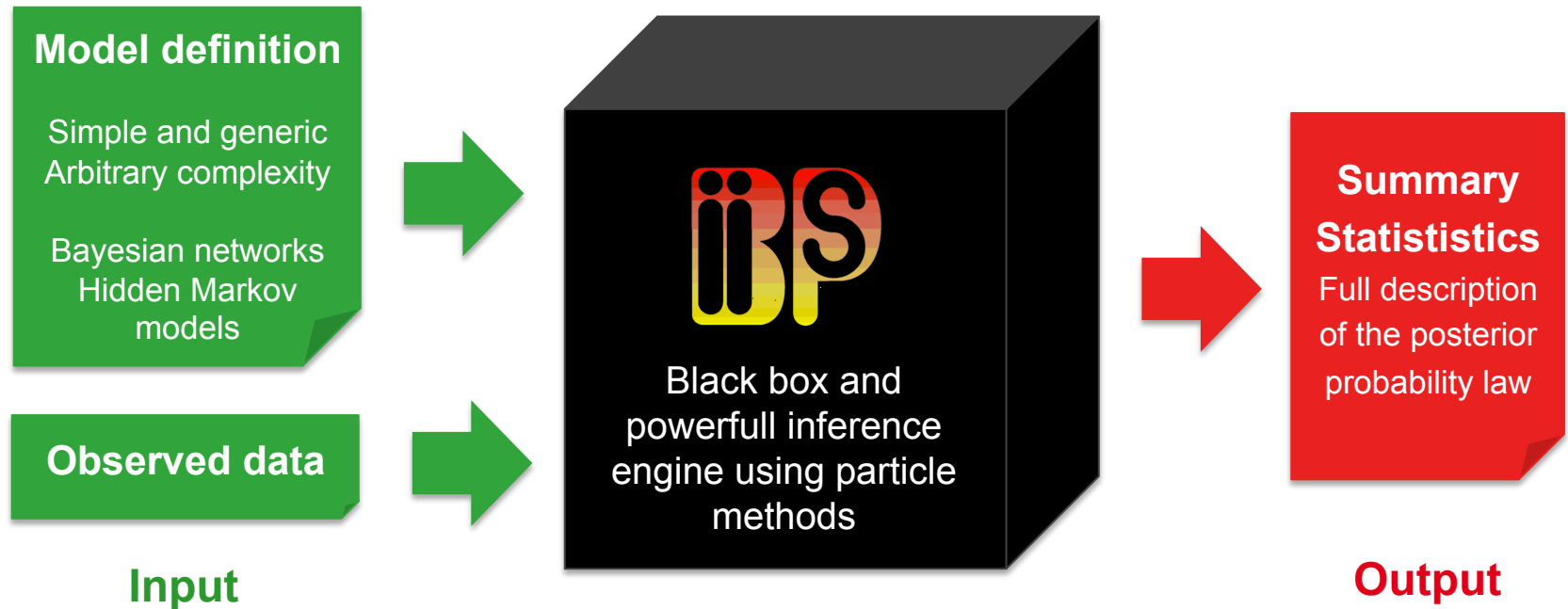
The software BiiPS

A new software for Bayesian inference using interacting particle systems



*Design and development: **A. Todeschini***

*Free and open source software soon available for Windows, Linux and Mac OS
at **<http://biips.gforge.inria.fr>***



Financial models calibration

Stochastic volatility / Partial observations

```

1 # Stochastic volatility model
2
3 model
4 {
5   x[1] ~ dnorm(mean.x.init, prec.x.init)
6   for (t in 2:t.max)
7   {
8     x[t] ~ dnorm(x[t-1], prec.x)
9     y[t] ~ dnorm(0, exp(-x[t]))
10  }
11 }

```

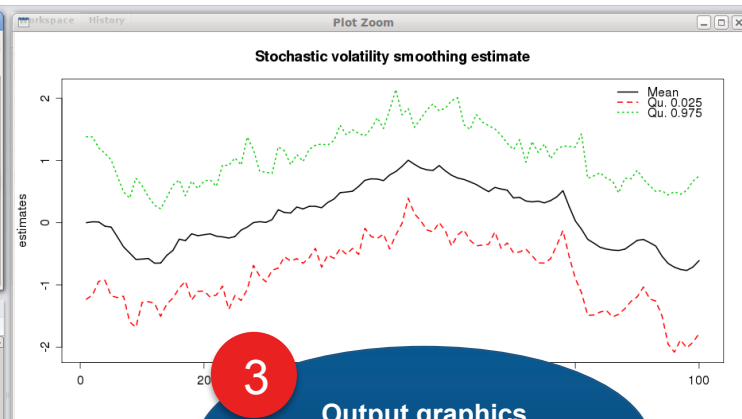
1 Model
Simple definition in
BUGS language

```

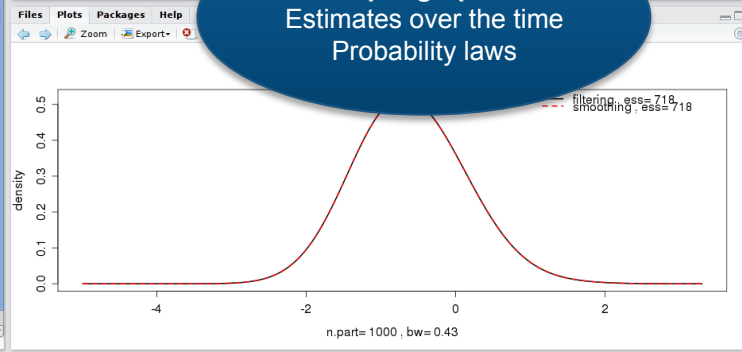
6 prec.x.init <- 1e-1;
7 prec.x <- 10
8 y <- scan("data/stoch_volatility.txt", n)
9
10 # model
11 biips <- biips.model("models/stoch_volatility",
12                    "stoch_volatility_demo.bug",
13                    # run biips: smc.samples
14                    n.part <- 1000
15                    out.biips <- smc.samples(biips, "x",
16                                             type=c("filtering", "smoothing"),
17                                             n.part=n.part, backward=FALSE)
18
19 # summary
20 s <- summary(out.biips[["x"]], fun=c("mean", "quantiles"), probs=c(.025, .975))
21
22 # plots
23 plot(density(out.biips[["x"]], adjust=2))
24 plot(s$filtering, xlab="time", main="Stochastic volatility filtering estimate")
25 plot(s$smoothing, xlab="time", main="Stochastic volatility smoothing estimate")
26
27 # diagnostic
28 print(diagnostic(out.biips[["x"]]))

```

2 R script
Few powerfull
command lines



3 Output graphics
Estimates over the time
Probability laws



Future works: Matlab interface, Parallelization and more...

3

More finance applications

Pricing financial products

European barrier options, Asian options, American options, ...

Mathematical tools:

- Optimal control
- Particle approximation of Snell envelope

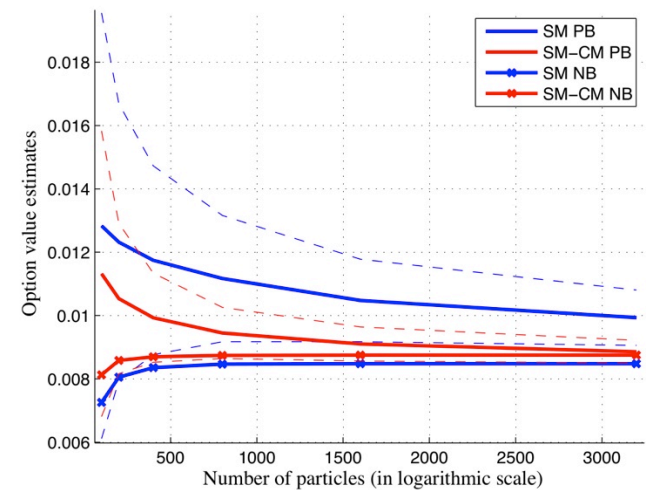
References:

- R. Carmona, P. Del Moral, P. Hu and N. Oudjane (eds.), Numerical Methods in Finance, to appear, Springer-Verlag.
- R. Carmona, P. Del Moral, P. Hu and N. Oudjane, "An introduction to particle methods in finance" [43 pages], to appear in Numerical Methods in Finance.
- P. Del Moral, P. Hu and N. Oudjane, "Snell Envelope with small probability criteria" [22 pages], preprint inria-00507794 [submitted], 2010.
- P. Del Moral, P. Hu, N. Oudjane and B. Rémillard, "On the Robustness of the Snell Envelope" [40 pages], SIAM J. Finan. Math., Vol. 2, pp. 587–626, 2011.

Collaboration:

EDF R&D

Pricing efficiency improved by using advanced particle estimation



Monte Carlo estimates vs. advanced particle estimates

Payoff	K	$d = 1$	$d = 2$	$d = 3$	$d = 4$	$d = 5$
Geometric Put	0.95	1 (1%)	1 (3%)	1 (6%)	1 (9%)	1 (10%)
	0.85	5 (2%)	8 (6%)	6 (11%)	4 (14%)	3 (14%)
	0.75	18 (6%)	28 (11%)	18 (17%)	16 (18%)	11 (16%)
Arithmetic Put	0.95	1 (1%)	3 (2%)	3 (7%)	4 (13%)	5 (18%)
	0.85	5 (2%)	13 (6%)	24 (19%)	56 (24%)	100 (20%)
	0.75	18 (6%)	71 (15%)	363 (14%)	866 (16%)	– (–)

Ratios of estimated MC results vs. advanced particle results

Hedging

Greeks calculation

Mathematical tools:

- Sequential Monte Carlo particle methods
- Particle gradient models

References:

- R. Carmona, P. Del Moral, P. Hu and N. Oudjane (eds.), Numerical Methods in Finance, to appear, Springer-Verlag.
- R. Carmona, P. Del Moral, P. Hu and N. Oudjane, “An introduction to particle methods in finance” [43 pages], to appear in Numerical Methods in Finance.
- P. Del Moral, P. Hu and L. Wu, On the concentration properties of Interacting particle processes, Foundations and Trends in Machine Learning, vol. 3, nos. 3–4, pp. 225–389, 2012.



Credit Risk

Default Probability

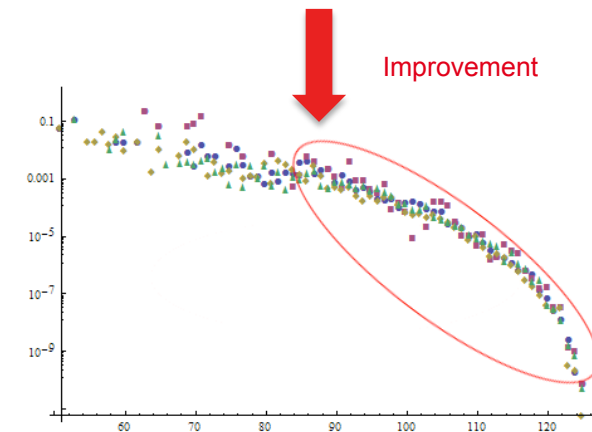
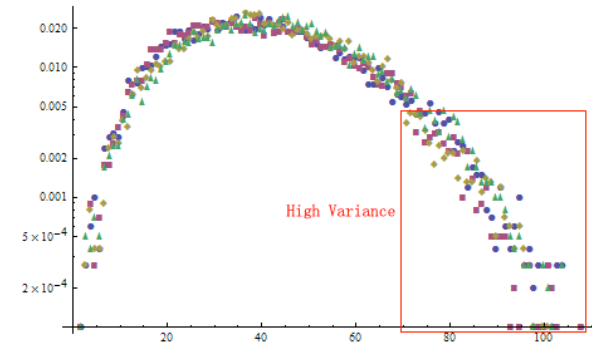
Mathematical tools:

- Rare event simulation
- Particle default tree models

References:

- R. Carmona, P. Del Moral, P. Hu and N. Oudjane, “An introduction to particle methods in finance” [43 pages], to appear in Numerical Methods in Finance.
- R. Carmona and S. Crépey. Importance Sampling and Interacting Particle Systems for the Estimation of Markovian Credit Portfolios Loss Distribution. International Journal of Theoretical and Applied Finance, vol. 13, No. 4 (2010) 577 – 602.
- R. Carmona, J.-P. Fouque and D. Vestal. Interacting Particle Systems for the Computation of Rare Credit Portfolio Losses. Finance and Stochastics, vol. 13, no. 4, 2009 pp. 613-633 (2009).

Default distribution (log-scale)
for a portfolio of 125 stocks



Rare credit portfolio losses are better captured with advanced particle estimation

Portfolio optimization

*Maximizing expected utility function
of the terminal wealth*



Mathematical tools:

- Abstract solutions with backward SDEs

References:

- Y. Hu, P. Imkeller, M. Müller, "Utility maximization in incomplete markets." *Ann. Appl. Probab.*, 15(3): 1691-1712, 2005.
- Y. Hu, P. Imkeller, M. Müller, "Partial equilibrium and market completion." *Int. J. Theor. Appl. Finance*, 8(4): 483-508, 2005.
- U. Horst, Y. Hu, P. Imkeller, A. Réveillac, J. Zhang, "Forward-backward systems for expected utility maximization", Preprint, 2011.

- Numerical simulation: time discretization of the solution of quadratic backward SDEs

References:

- A. Richou, "Numerical simulation of BSDEs with drivers of quadratic growth." *Ann. Appl. Probab.*, 21(5): 1933-1964.
- A. Richou, "Markovian quadratic and superquadratic BSDEs with an unbounded terminal condition." Preprint, 2012.
- A. Richou, "Etude théorique et numérique des équations différentielles stochastiques rétrogrades", PhD thesis, Univ. Rennes 1, 2010.

Collaborations

Oxford University, A. Doucet, T. Lyons

Cambridge University, S. S. Singh

Princeton University, R. Carmona

University of California Santa Barbara, J. P. Fouque

Singapore University, A. Jasra

SCIRO Sydney, P. Shevchenko

University College London, G. Peters

Imperial College, D. Crisan

Columbia University, J. Blanchet, Ph. Protter

Ecole Polytechnique, E. Gobet, N. Touzi

EDF R&D, N. Oudjane

Conference information

Workshop on Sequential Monte Carlo methods and Efficient simulation in Finance.

ALEA – CMAP. Ecole Polytechnique, Paris October 8-12th, 2012